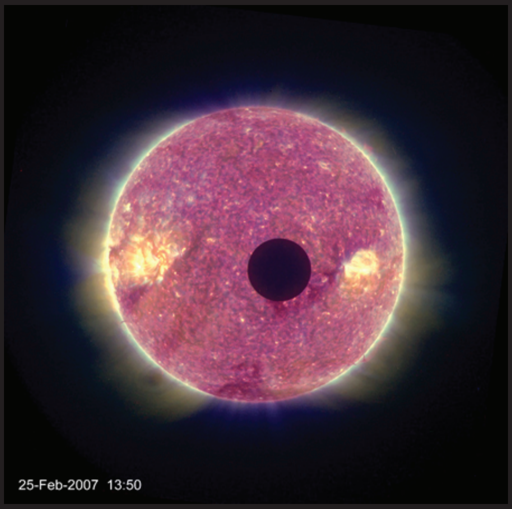
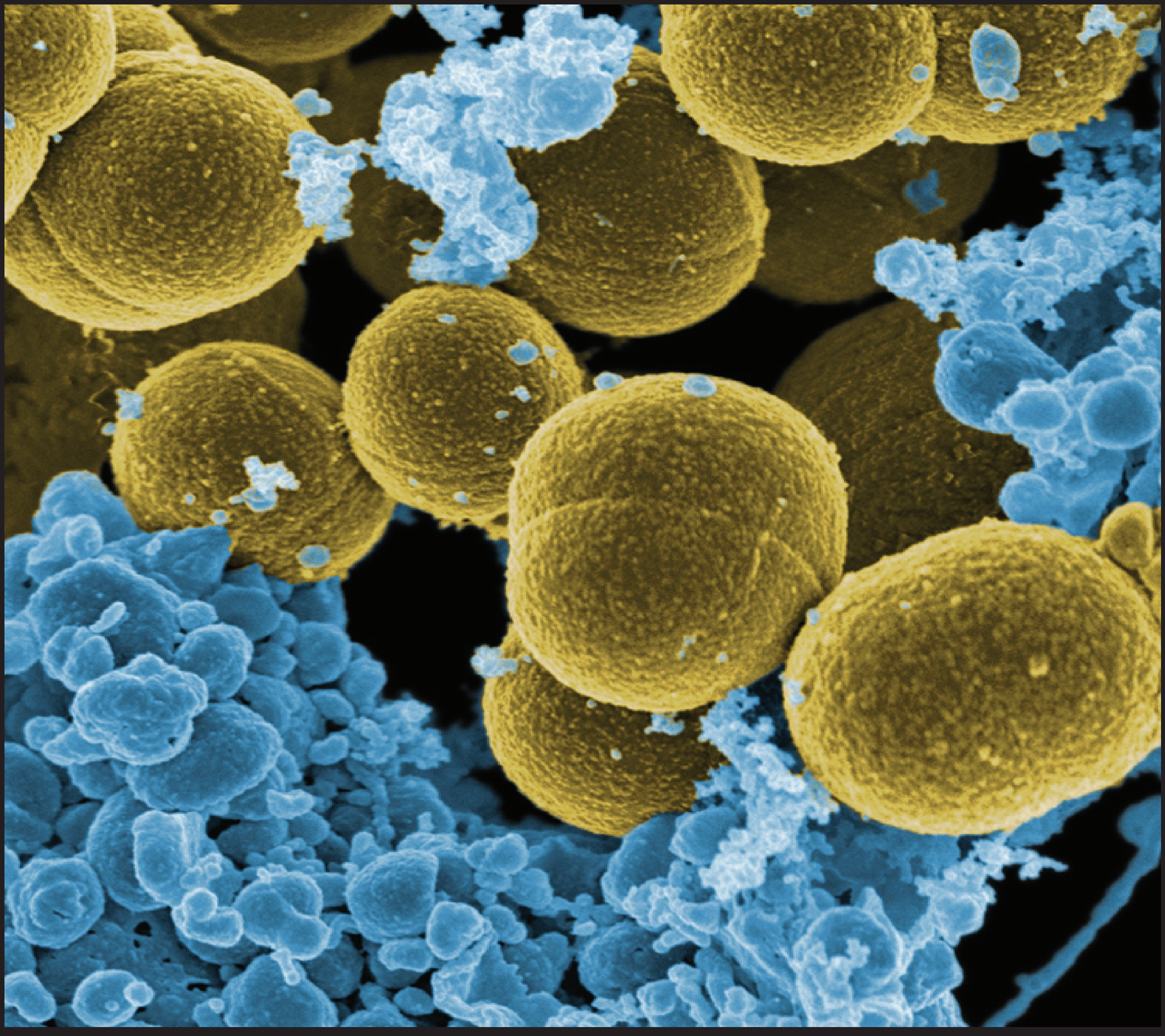
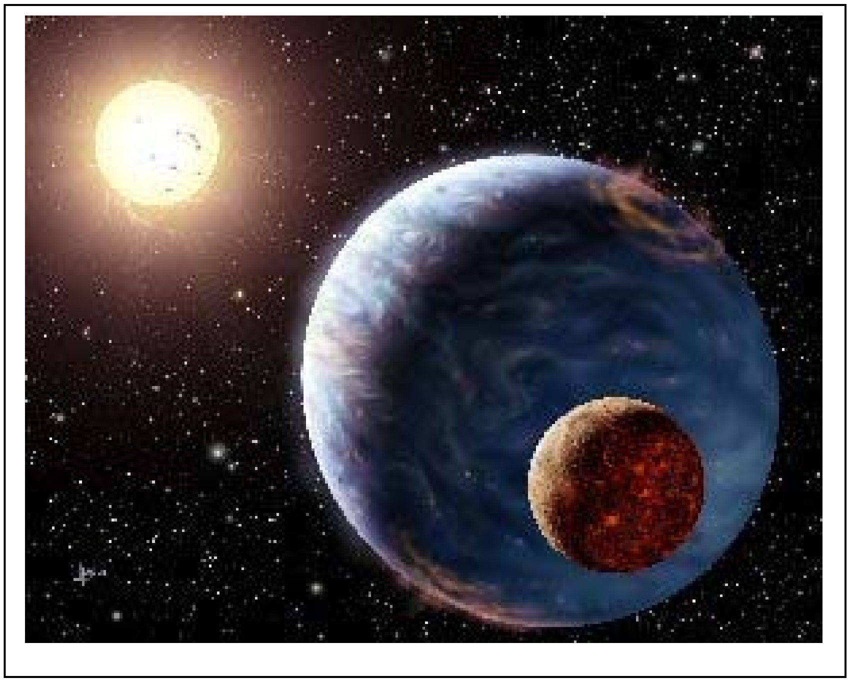
National Aeronautics and Space Administration





Astrobiology Math

Astrobiology Math



Mathematical problems featuring astrobiology applications

Dr. Sten Odenwald
Catholic University of America / NASA
Sten.F.Odenwald@nasa.gov

This collection of activities is based on a weekly series of space science problems intended for students looking for additional challenges in the math and physical science curriculum in grades 6 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

Acknowledgments:

We would like to thank Ms. Daniella Scalice for her boundless enthusiasm in the review and editing of this resource. Ms. Scalice is the Education and Public Outreach Coordinator for the NASA Astrobiology Institute (NAI) at the Ames Research Center in Moffett Field, California.

We would also like to thank the team of educators and scientists at NAI who graciously read through the first draft of this book and made numerous suggestions for improving it and making it more generally useful to the astrobiology education community.

For more weekly classroom activities about astronomy and space visit the Space Math@ NASA website, http://spacemath.gsfc.nasa.gov

Image Credits:

Front Cover Top)

Created under an Education and Public Outreach grant, NNH08CD59C administered through the NASA Science Mission Directorate.

Alignment with Standards

AAAS Project:2061 Benchmarks

(9-12) - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

NCTM:"Principles and Standards for School Mathematics"

Grades 6–8 :

- work flexibly with fractions, decimals, and percents to solve problems;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; .
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- model and solve contextualized problems using various representations, such as graphs, tables, and equations.
- use graphs to analyze the nature of changes in quantities in linear relationships.
- understand both metric and customary systems of measurement;
- understand relationships among units and convert from one unit to another within the same system;

Grades 9–12 :

- judge the reasonableness of numerical computations and their results.
- generalize patterns using explicitly defined and recursively defined functions;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- draw reasonable conclusions about a situation being modeled.

Mathematics Topic Matrix

Topic	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Inquiry																															
Technology, rulers																											X				
Numbers, patterns, percentages		X	X		X	X	X	X	X			X		X								X	X							X	
Averages																															
Time, distance, speed																															
Areas and volumes											X						X		X					X			X		X		X
Scale drawings																											X				
Geometry																															
Probability, odds																															
Scientific Notation						X	X						X				X	X	X				X		X					X	
Unit Conversions																X						X				X					
Fractions																															
Graph or Table Analysis	X		X	X				X		X			X	X	X											X		X	X		X
Pie Graphs																															
Linear Equations																X															
Rates & Slopes																															
Solving for X											X					X												X			
Evaluating Fns															X			X		X	X				X						
Modeling																															
Trigonometry																															
Logarithms				X																											
Calculus																		X													

Mathematics Topic Matrix

Topic	Problem Numbers																													
	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
Inquiry																															
Technology, rulers	X								X		X		X							X							X				
Numbers, patterns, percentages			X	X	X	X														X	X	X	X	X	X	X	X		X		
Averages																											X				
Time, distance, speed																															
Areas and volumes	X	X										X		X	X	X															
Scale drawings	X								X		X		X							X	X	X	X	X	X	X	X				
Geometry																															
Probability, odds																															
Scientific Notation										X		X		X	X	X	X														X
Unit Conversions																															
Fractions																															
Graph or Table Analysis		X		X		X		X			X																			X	
Pie Graphs																															
Linear Equations																															
Rates & Slopes			X																												
Solving for X							X			X																					
Evaluating Fns							X	X		X				X		X	X	X	X									X		X	X
Modeling								X		X						X	X	X	X									X	X	X	X
Trigonometry																															
Logarithms																															
Calculus																															X

Mathematics Topic Matrix

Topic	Problem Numbers												
	63	64	65	66	67	68	69	70	71	72	73	74	75
Inquiry													
Technology, rulers													
Numbers, patterns, percentages	X			X	X		X			X	X	X	X
Averages													
Time, distance, speed						X							
Areas and volumes													
Scale drawings													
Geometry													
Probability, odds							X			X	X	X	X
Scientific Notation			X	X		X		X					
Unit Conversions													
Fractions													
Graph or Table Analysis		X	X						X				
Pie Graphs													
Linear Equations													
Rates & Slopes													
Solving for X													
Evaluating Fns			X	X	X			X				X	
Modeling				X	X			X					
Trigonometry													
Logarithms									X				
Calculus													

Additional Titles in this Series

Black Holes (2008) 11 Problems An introduction to the basic properties of black holes using elementary algebra and geometry. Students calculate black hole sizes from their mass, time and space distortion, and explore the impact that black holes have upon their surroundings.

Image Scaling (2008) 11 Problems Students work with a number of NASA photographs of planets, stars and galaxies to determine the scales of the images, and to examine the sizes of various features within the photographs using simple ratios and proportions.

Radiation (2007) 19 Problems An introduction to radiation measurement, dosimetry and how your lifestyle affects how much radiation your body absorbs.

Solar Math (2008) 15 Problems Exploring solar storms and solar structure using simple math activities. Calculating speeds of solar flares from photographs, and investigating solar magnetism.

Lunar Math (2008) 17 Problems An exploration of the moon using NASA photographs and scaling activities. Mathematical modeling of the lunar interior, and problems involving estimating its total mass and the mass of its atmosphere.

Magnetic Math (2009) 37 Problems Six hands-on exercises, plus 37 math problems, allow students to explore magnetism and magnetic fields, both through drawing and geometric construction, and by using simple algebra to quantitatively examine magnetic forces, energy, and magnetic field lines and their mathematical structure.

Earth Math (2009) 46 Problems Students explore the simple mathematics behind global climate change through analyzing graphical data, data from NASA satellites, and by performing simple calculations of carbon usage using home electric bills and national and international energy consumption.

Electromagnetic Math (Draft:2010) 84 Problems Students explore the simple mathematics behind light and other forms of electromagnetic energy including the properties of waves, wavelength, frequency, the Doppler shift, and the various ways that astronomers image the universe across the electromagnetic spectrum to learn more about the properties of matter and its movement.

Space Weather Math (Draft:2010) 96 Problems Students explore the way in which the sun interacts with Earth to produce space weather, and the ways in which astronomers study solar storms to predict when adverse conditions may pose a hazard for satellites and human operation in space. Six appendices and an extensive provide a rich 150-year context for why space whether is an important issue.

Introduction to Astrobiology

Not long ago, the thought of actually searching for life beyond Earth was considered pure speculation at best, and often tedious and fearsome science fiction at worst. Pulp science fiction authors began writing about human encounters with alien life in the 1920's, and this steady diet of fantastic story-telling led to some simply dreadful movies during the last half of the 20th century. Such infamous movies as 'Alien' (1979) portrayed the horrific result of human-

alien contact. In stark contrast, '*2001: Space Odyssey*' (1969) and 'Contact' (1997) present a much more hopeful consequence. Not surprisingly, perhaps, the later movies were written by Arthur C. Clarke and astronomer Carl Sagan. In fact, many of the modern generations of scientists and astronomers grew-up with these stories, and in some cases the stories became the stimulus for many professional careers in these fields.

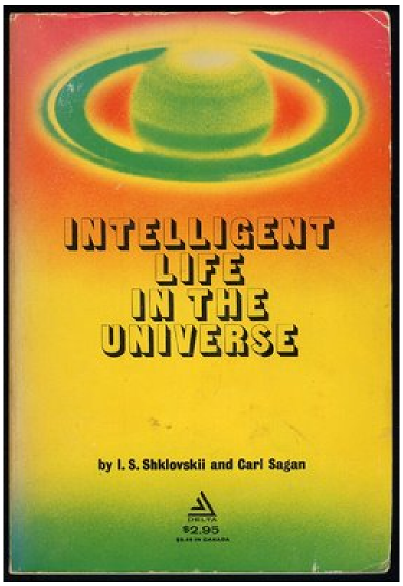
Perhaps because of the 'bad press' that aliens and 'UFOs' had acquired for most of the 20th Century, it was often considered something of a professional death sentence for a respectable astronomer or biologist to dwell too long on alien life, beyond a speculative comment presented largely in private to students in undergraduate astronomy classes. However, by the 1960's, this reticence to confront alien life waned as the Soviet and American space programs began. Was there life in space? Increasingly the answer became 'Yes' as more humans ventured into the heavens one rocket launch at a time. And then something else happened.

In 1960, radio astronomer Frank Drake conducted the first study of a promising the nearby star, Epsilon Eridani, and concluded that there were no interesting radio signals from that star at the present time. His unfunded and part-time studies, called Project OZMA, led to a proposal for a quantitative way of estimating the probability of life in our galaxy, hereafter called the Drake Equation.

$$N = N^{*} f_p N_e f_l f_i f_c L$$

where N* = the number of stars born each year in the Milky Way, f_p is the fraction of stars with planets; n_e is the average number of habitable planets per star; f_l is the fraction of habitable planets that have life; f_i is the fraction that develop intelligent life; f_c is the fraction of civilizations that develop interstellar communication, and L is the lifetime of detectable intelligent emission. Each of these factors was a crude attempt at assessing, step by step, the conditions that had to be satisfied in order for there to be intelligent life in the Milky Way today, whit whom we could at least in principle communicate using radio waves.

In 1968, an unknown assistant professor at Cornel University by the name of Carl Sagan, teamed up with a Soviet astronomer Iosef Shklovskii, and together they wrote a path-breaking book *'Intelligent Life in the Universe*'. It was published by Dell Publishing: hardly an academic publisher, and in fact, a major publisher of pocketbook science fiction novels! The book was largely a translation of Shklovskii's book 'Universe, Life, Mind' which had already been published in the Soviet Union in 1963 in commemoration of the launch of Sputnick. It quickly became required reading in many undergraduate astronomy courses. This was not, however, the first time that astronomers had waded into the messy business of speculating about life elsewhere in the universe, but it was most certainly the most mathematically, biologically, and astrophysically-based argument for how to critically think about the subject.



The bottom line, according to Sagan, was that there could me over one million intelligent civilizations in the Milky Way at the present time, and able to communicate via radio signals. We should be listening for them by conducting the proper long-term monitoring studies. Within a decade, in 1971, a new more aggressive 'SETI Program' (Search for Extra Terrestrial Intelligence) was spear-headed by Dr. Frank Drake and Bernard Oliver, and funded through a small grant by NASA. It would be called Project Cyclops and consist of 1,500 radio telescope dishes ,at a cost of $10 billion. Although it was not funded, the engineering details became the main-stay of future search programs and anticipated many of the challenges facing such a daunting program.

In 1979, Dr. Jill Tarter at UC Berkeley was able to set up a program called SERENDIP, which used its own independent hardware operating in parallel with funded astronomical observing programs at major radio observatories, to serendipitously hunt for artificial signals and signal patterns in time, using a large, portable, multi-channel receiver. This system was eventually superceded by 100,000-channel spectrum analyzers used by Dr. Paul Horowitz at Harvard University in the early 1980s. This system was in turn upgraded in Project META to million-channel analyzers in 1985, and finally the gargantuan Project Beta's billion-channel analyzers, which came on-line in 1995.

Throughout this period, Congress routinely ridiculed the entire concept of the search, and so federal funds through such agencies as the National Science Foundation and NASA could not be used in this way. However, that did not stop private and corporate donations which, not only wanted to support the interesting development of powerful spectrum analyzers, but enjoyed the notoriety of being at the forefront of scientific research.

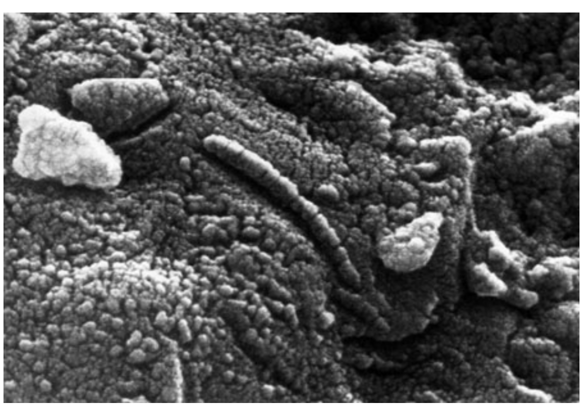
Currently, data from the spectrum analyzers is being processed through a massively distributed network of PCs under the program SETI@HOME. Now, everyone with a computer can run the SETI search program and process actual data as a background job or a screen-saver application!

Also, thanks to the financial support by Paul Allen, co-founder of Microsoft, a mini-Cyclops array of 350 dishes is being completed in Hat Creek, California .The Allen Telescope Array will be dedicated to SETI, and has already begun operating with 42 dishes installed. The array will span a frequency range from 500 megaHertz to 11 gigaHertz, and cover a sky area 5 times the size of the full moon, containing 100,000's of stars, searched simultaneously across billions of individual channels in this frequency range.

At the same time these programs were supporting the serious search for 'ET', during the 1990's a series of discoveries made the news, and even reached as far as the White House.

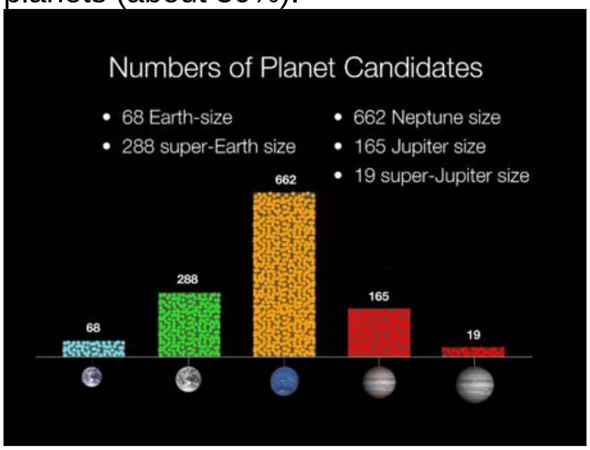
The first non-stellar bodies orbiting another star were discovered in 1992 by Aleksander Wolszczan and Dale Frail orbiting the millisecond pulsar PSR 1257+12. Two of the four planets had masses of 2 and 4 times that of Earth. These were the first two extrasolar planets confirmed to be discovered, and thus the first multi-planet extrasolar planetary system discovered.

Then, in 1996, a group of scientists led by David McKay of NASA's Johnson Space Center published an article in the August 16, 1996 issue of *Science* magazine announcing the discovery of evidence for primitive bacterial life on Mars. An examination of a meteorite found in Antarctica, Allan Hills 84001, and believed to be from Mars shows: 1) hydrocarbons which are the same as breakdown products of dead micro-organisms on Earth, 2) mineral phases consistent with by-products of bacterial activity, and 3) tiny carbonate globules which may be microfossils of the primitive bacteria, all within a few hundred-thousandths of an inch of each other.



Although the ALH84001 discovery remains highly controversial, the involvement of NASA in the study led to a fundamental shift in the governmental discussion of ET. NASA re-defined some of its new goals to include a new *'NASA Astrobiology Institute'* (http://astrobiology.nasa.gov/) at the NASA facility at the Ames Research Center, Moffett Field, California, to be directed by Dr. Ed Weiler; the former Head of NASA's Science Mission Directorate in Washington, D.C. The research now includes the intensive study of 'extremophile' bacteria, and developing new methods for detecting extraterrestrial life environments. These now include sub-surface regions of Jupiter's moon Europa, the search for water on Mars, and ice deposits in shadowed craters on the Moon.

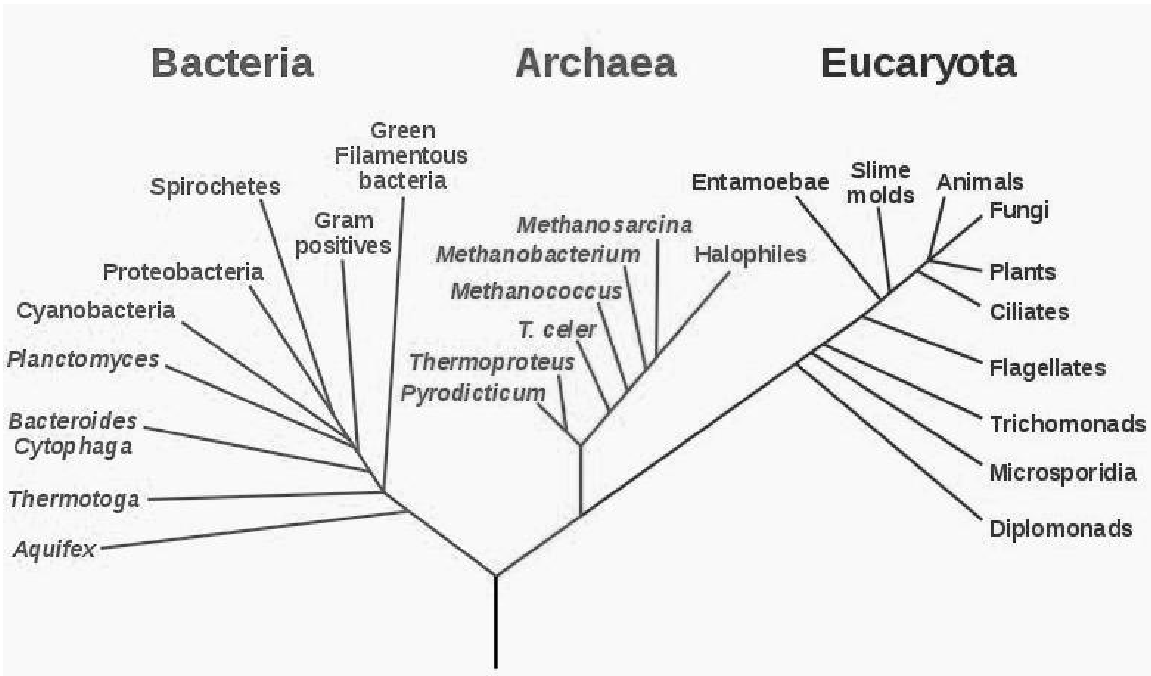
In 1995, Michael Mayor and Didier Queloz at the University of Geneva used a highly-sensitive spectroscopic technique to measure the speed of the sun-like star 51 Pegasi located 51 light years from Earth. Their discovery of a Jupiter-sized planet orbiting that star began the current era of planet searching. Their work was expanded upon by Geoffrey Marcy, Debra Fisher and Paul Butler at San Francisco State University, using their own high-resolution spectrometer at the Lick Observatory. By 2011, the current count for extra-solar planets, now called simply exoplanets' is 548. Extensive studies of their orbits, sizes and other factors has now established, for the first time, the values for many of the 'astronomical' factors in the Drake Equation, such as the percentage of stars with planets (about 30%).



On March 7, 2009, NASA launched the Kepler Mission; a 1-meter telescope equipped with an ultra sensitive CCD-based photometer. The goal of the 4-year mission was to measure the minute-to-minute brightness changes of 157,000 stars in the direction of the constellation Cygnus. The stars were all known to be sun-like, and the intent was to determine from the planetary transits just how many stars had planets, and what fraction of those planets were earth-sized and orbited within the habitable zones of their stars. In these Habitable Zones, planetary surface temperatures were in the range from 273 to 373 Kelvin so that water could remain in liquid form. By 2011, the mission had released a list of 1235 candidate planets, of which 68 were found to be Earth-sized, and 5 of those were located in their Habitable Zones. During the next 3 years, a longer time baseline will capture still more transiting planets, but with orbit periods up to several years in length, and even more Earth-like in their locations from their stars.

In the breathtaking 50 years since Frank Drake and Carl Sagan first made the search for ET a legitimate scientific undertaking, we have explored Earth-based life that lives in boiling water, and discovered thousands of planets beyond our solar system. One can only wonder what new discoveries lie just around the next corner!

The Tree of Life and the Most Primitive Ancestor



Modern techniques based on detailed genetic cell studies have led to a 'Tree of Life'. The tree consists mostly of bacterial life forms with Animals, Plants and Fungi appearing late in the evolutionary process about 550 million years ago. Common similarities among all known terrestrial organisms living today leads to a single 'Last Universal Ancestor' at the base of this tree, which lived about 3.5 to 3.8 billion years ago.

Problem - Use a Venn Diagram to find the common properties of the three hypothetical life forms listed below, and describe what properties the Last Universal Ancestor may have had, and which properties are probably adaptations.

Heebie : DNA replication; three legs; nocturnal; cells with bi-lipid membrane structure; mitochondria; nervous system; lives in water; oxygen respiration; water and carbon based; hairless; mass range in grams $10 < M < 1000$; cells have nuclei; sexual reproduction; has no teeth; eats plants and animals

Ogryte: No legs; photosynthesis; DNA replication; active in daytime; cells with bi-lipid membrane structure; lives on land; mitochondria; nervous system; oxygen respiration; water and carbon-based; hairless; mass range in grams $20 < M < 100$; cells have nuclei; sexual reproduction; large but few teeth; eats animals

Rhymba: One Leg; nocturnal; cells with bi-lipid membrane structure; mitochondria; oxygen respiration; lives on land; water and carbon-based; surface hair; mass range in grams $50 < M < 70$; cells have nuclei; sexual reproduction; small but many teeth; eats plants and animals.

Table of properties. Highlighted entries indicate common properties.

Heebie	Ogryte	Rhymba
DNA replication	DNA replication	DNA replication
3 legs	No legs	One leg
nocturnal	Daytime active	nocturnal
Cells have bilipid membrane	Cells have bilipid membrane	Cells have bilipid membrane
mitochondria	mitochondria	mitochondria
Nervous system	Nervous system	
Oxygen respiration	Oxygen respiration	Oxygen respiration
Water and carbon based	Water and carbon based	Water and carbon based
hairless	hairless	hairy
Mass 10 < M < 1000	Mass 20 < M < 100	Mass 50 < M < 70
Cells have nuclei	Cells have nuclei	Cells have nuclei
Sexual reproduction	Sexual reproduction	Sexual reproduction
Lives in water	Lives on land	Lives on land
No teeth	Large but few teeth	Small but many teeth
Eats plants and animals	Eats animals	Eats plants and animals

The Last Universal Ancestor (LUA) had the following properties:
1 - The LUA was based upon water and carbon;
2 - Its cells had walls based upon a bilipid membrane structure;
3 - the cells had nuclei;
4 - DNA replication was used, and
5 - the LUA used sexual rather than asexual reproduction to copy itself;
6 - the cells possessed mitochondria;
7 - the LUA respired oxygen;
8 - the LUA probably ate both plants and animals;
9 - the mass range that is consistent with the three constraints is 50 < M < 70 grams.

All of the other factors that are not common are probably produced by speciation and adaptation as the LUA entered different ecological niches.

The Venn Diagram should have three intersecting rings. In the region of intersection should be the 9 common attributes described in the paragraph.

We are all familiar with what is popularly called the 'Age of the Dinosaurs', but in the study of terrestrial life, there were many important ages that predated the dinosaurs of the Cretaceous era 65 million years ago. Geologists and paleontologists identify over a dozen important eras since the formation of Earth 4.6 billion years ago. Life on other worlds, if it exists, may have evolved through similar kids of eras during the life span of its planet or star. In the following list, time is indicated in billions of years (Gy).

Hadean Era (4.65 to 3.8) - Formation of Earth's surface; Massive asteroidal impacts. Formation of Moon. Deposition of oceans. Appearance of freshwater on land.

Archaean Era (3.4 - 2.5) - First evidence of single-celled organisms (3.8). Photosynthesis (3.8); First microfossils (3.6); First oxygen-producing bacteria (3.6); First stromatolite fossil mats (3.0);

Proterozoic Era (2.5Gy - 550 My) - Atmosphere becomes oxygenic (2.1); active mountain building; Protists appear as first complex single-celled life (1.8) ; Multi-celled eukaryotes (1.0); Snowball Earth (850My); Ediacaran life forms flourish (600 My)

Phanerozoic Era (550 My to 65 My) - Major diversification of multi-cellular life in oceans, land and air

Cenozoic Era (65 My to Now) - End of dinosaur megafauna (65 My); Rise of mammalian life forms; human evolution (5 My); Modern humans appear (50,000 yrs)

Given the durations of the various eras as noted above, and assuming that you selected a planet similar to Earth in age, mass, size and average temperature, what might you estimate for the following percentages expressed as percentages relative to the age of the planet:

Problem 1 - The percentage of time that the planet has life of any type?

Problem 2 - The percentage of time that only bacterial life (single celled) is present?

Problem 3 - The percentage of time that the life forms are more complex than bacteria?

Problem 4 - The percentage of time that the life forms are large and at least as intelligent as mammals?

Problem 5 - The percentage of time that the life forms are as intelligent as modern humans?

Problem 6 - You find 100,000 Earth-like planets that are 4.6 billion years old. How many will have life as intelligent as modern humans?

Answer Key

Problem 1 - The percentage of time that the planet has life of any type?

Answer: 100% x (3.8 billion years / 4.6 billion years) = **83%**

Problem 2 - The percentage of time that only bacterial life (single celled) is present?

Answer: During the Proterozoic Era, multi-cellular life appeared 1 billion years ago, so only bacterial life existed from 3.8 to 1.0 billion years ago or 2.8 billion years, then

100% x (2.8 billion years / 4.6 billion years) = **60%**

Problem 3 - The percentage of time that the life forms are more complex than bacteria?

Answer: During the Proterozoic Era, multi-cellular life appeared 1 billion years ago, and existed to the present, so this period lasted 1 billion years,

100% x (1.0 billion years / 4.6 billion years) = **22%**

Problem 4 - The percentage of time that the life forms are large and at least as intelligent as mammals?

Answer; This period lasted from 65 million years ago to the present, so

100% x (65 million years / 4.6 billion years) = **1.4%**

Problem 5 - The percentage of time that the life forms are as intelligent as modern humans?

Answer:
100% x (50,000 years / 4.6 billion years) = **0.001%**

Problem 6 - You find 100,000 Earth-like planets that are 4.6 billion years old. How many will have life as intelligent as modern humans?

Answer: N = 100000 x (0.001% / 100%) = **1 planet**

When we think about living organisms, we usually think in terms of a classification scheme that distinguishes living things according to their physical properties. A single Monarch butterfly is distinct from other kinds of butterflys in other species with which it cannot (or will not) mate, and can be classified as follows:

Kingdom	*Animalia*
Phylum	*Arthropoda*
Class	*Insecta*
Order	*Lepidoptera*
Family	*Papilionoidea*
Genus	*Danaus*
Species	*D. Plexippus*

When thinking about extra terrestrial life, it is hard to imagine that similar kinds of classification schemes would not appear to distinguish all of the different attributes of alien life forms. Scientists have estimated the number of distinct non-insect, animal Orders (Birds, Reptiles, Mammals, Amphibians) that have appeared in the fossil record during the last 500 million years. One such counting is shown in the table below *(P. Signor in 'Biodiversity in Geologic Time', American Zoologist, vol. 34, pp 23-32; 1994)*:

Era	**Time**	**Orders of Animalia**
Devonian	400	7
Mississippean	345	5
Pennsylvanian	310	20
Permian	270	20
Triassic	230	15
Jurassic	170	25
Cretaceous	100	25
Paleogene	50	40
Current	0	65

Problem 1 - What is the average rate of change of the number of Orders per million years over the 600 million year span of time between the Devonian and the Neogene Eras on Earth?

Problem 2 - If there are on average 200 species per Order, what is the average speciation rate (new species per million years) during the last 400 million years of Earth history?

Problem 3 - Suppose a planet was discovered where the 'Devonian Era' started 3 billion years after its formation, and the planet was now 5 billion years old. If speciation occurred at the terrestrial rate, how many species might exist in this planet's biosphere today?

Problem 1 - What is the average rate of change of the number of Orders per million years over the 600 million year span of time between the Devonian and the Neogene Eras on Earth?

Answer: The rate of change is just the change in the number of Orders divided by the amount of time so O = (65 - 7) Orders / 400 = **0.145 Orders per million years**.

Problem 2 - If there are on average 200 species per Order, what is the average speciation rate (new species per million years) during the last 400 million years of Earth history?

Answer: S = 0.145 Orders/Myr x (200 Species/Order) = **29 Species/Million years**

Problem 3 - Suppose a planet was discovered where the 'Devonian Era' started 3 billion years after its formation, and the planet was now 5 billion years old. If speciation occurred at the terrestrial rate, how many species of animals might exist in this planet's biosphere today?

Answer: The growth of animal Orders has been going on for 2 billion years starting with 7 Orders so the current number would be O = 7 + 0.145 Orders/Myr x (2,000 Myrs) = 297 Orders. Since there are 200 Species per Order, there would be about 297 x 200 = **59,400 species of animals** (amphibians, reptiles, birds, mammals).
By comparison, there are about 13,000 species of these animals on Earth.

Note: We have only included animals equivalent to terrestrial birds, reptiles, mammals and amphibians. On Earth, these only account for 3% of all species in the Animal Kingdom because the rest are Invertebrates (especially insects). An even larger share of species are bacterial. In terms of probability, it is far more likely that a selected planet would be populated by insects (30 million species on Earth) than by vertebrates (1 million species on Earth). The question is whether there are enough ecological niches for 60,000 species of complex, vertebrate animals to occupy without rapidly going extinct. This is where mathematical modeling begins to break down because now the model has to include the presence of new ecological niches on alien worlds that were not found on Earth during the last 400 million years.

Small things are easier to make than large things, and they are also more numerous, so if we are trying to understand how life arises, it helps to find the smallest example of a 'living thing'. In that search, scientists have found over the years including bacteria, viruses, and prions. This list includes objects that possess some, but not all, of the most basic attributes of life: Reproduction, energy consumption and respiration.

The table below gives the sizes in nanometers (nm) of the known living and non-living objects. It also gives the number of genes, and the number of nucleotides in the DNA (or RNA) molecule.

Name	Size (nm)	State	Structure	Number of Nucleotides (in thousands)	Number of Genes
Porcene Circovirus	17	Non living	RNA	1.8	2
Hepatitus-B	42	Non living	RNA	3.0	4
Rous sarcoma Virus	80	Non living	RNA	3.5	3
Lambda bacteriophage	100	Nonliving	DNA	48	30
T4 bacteriophage	150	Nonliving	DNA	169	160
Mycoplasma Gen.	250	Living	DNA	582	521
Nanoarchaeum Equit.	400	Living	DNA	491	590
Escherichia Coli	2000	Living	DNA	4,000	3,000

Problem 1 - The smaller a virus or living system is, the less information it can store. Assume that each of the systems in the above table can be approximated as a sphere with the indicated diameter. Create a graph that shows the volume, V, of the system in cubic nanometers (nm^3) versus the number, N, of nucleotides (in thousands). Because the numbers will be very large, the actual values plotted should be the base-10 logarithms of N and V.

Problem 2 – A life form is discovered that has about 1,000 nucleotides. What might be a plausible classification for this life form as living or non-living?

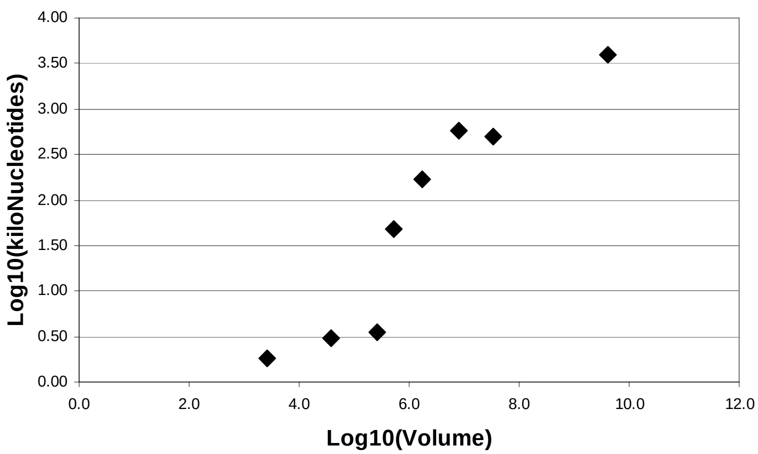
Problem 3 – A life form is discovered that has a volume of one trillion cubic nanometers. About how many nucleotides might this organism have?

Problem 4 – Suppose that a bacteriophage invaded an E. Coli bacterium and began reproducing until the bacterium wall ruptured. About what is the maximum number of bacteriophages that could be produced in the volume of the bacterium?

Answer Key

Problem 1 - Answer:

Name	D (nm)	N (thousands)	Volume Nm^3	Log(V)	Log(N)
Porcene Circovirus	17	1.8	2600	**4.3**	**0.26**
Hepatitus-B	42	3	39,000	**5.5**	**0.48**
Rous sarcoma Virus	80	3.5	268,000	**6.3**	**0.54**
Lambda bacteriophage	100	48	523,000	**6.6**	**1.68**
T4 bacteriophage	150	169	1,800,000	**7.2**	**2.23**
Mycoplasma Gen.	250	582	8,200,000	**7.8**	**2.76**
Nanoarchaeum Equit.	400	491	33,000,000	**8.4**	**2.69**
Escherichia Coli	2000	4,000	4.19E+09	**10.5**	**3.60**



Problem 2 – A life form is discovered that has about 1,000 nucleotides. What might be a plausible classification for this life form as living or non-living? Answer: **Non-living**.

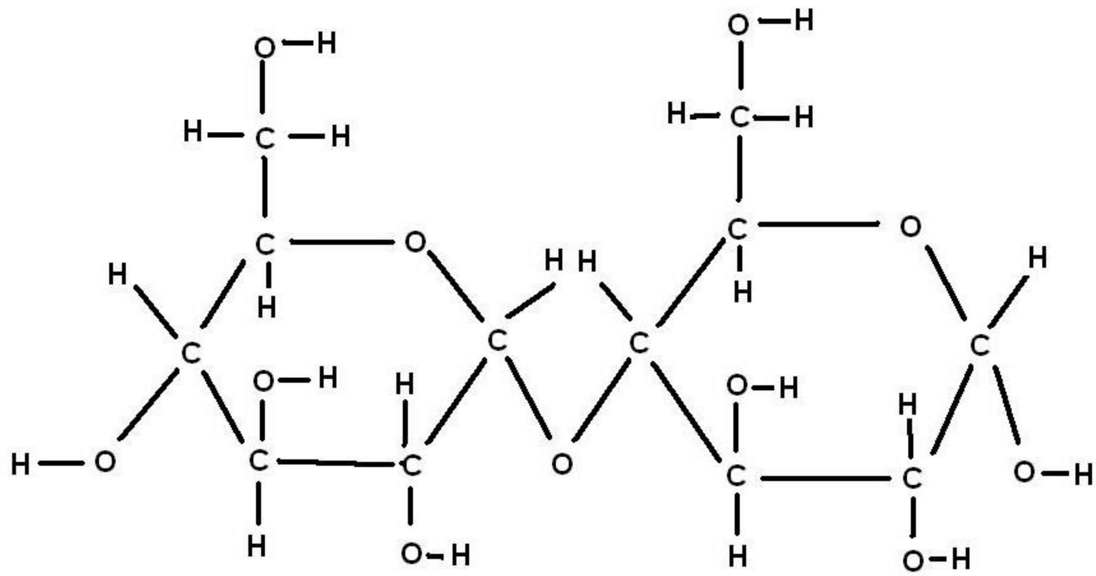
Problem 3 – A life form is discovered that has a volume of one trillion cubic nanometers. About how many nucleotides might this organism have?

Answer: A) From the graph, V = Log10(10^{12}) = 12.0 and so Log10 (N)= 4.0 so N = 10,000 kiloNucleotides or about **10 million nucleotides**.

Problem 4 – Suppose that a bacteriophage invaded an E. Coli bacterium and began reproducing until the bacterium wall ruptured. About what is the maximum number of bacteriophages that could be produced in the volume of the bacterium?

Answer: The bacteriophage volume is about 1.8 million nm^3 and the bacterium volume is about 4.2×10^9 nm^3, so the maximum number is about N = $4.2 \times 10^9 / 1.8 \times 10^6$ or about **2300 bacteriophages.** *In actuality, the process takes about 30 minutes and results in only about 100 bacteriophages.*

Atoms - How sweet they are!



Glucose is a very important sugar used by all plants and animals as a source of energy. Maltose is the next most complicated sugar, and is formed from two glucose molecules. The atomic ingredients of the maltose molecule is shown in the diagram above, which is called the structural formula for maltose. As an organic compound, it consists of three types of atoms: hydrogen (H), carbon (C), and oxygen (O).

Problem 1 - How many molecules does maltose contain of A) hydrogen? B) oxygen? C) carbon?

Problem 2 - What is the ratio of the number of hydrogen atoms to oxygen atoms?

Problem 3 - What fraction of all the maltose atoms are carbon?

Problem 4 - If the mass of 1 hydrogen atoms is 1 AMU, and 1 carbon atom is 12 AMU and 1 oxygen atom is 16 AMU, what is the total mass of one maltose molecule in AMUs?

Problem 5 - Write the chemical formula of maltose by filling in the missing blanks:

C__ H__ O_{11}

Answer Key

Problem 1 - How many molecules does maltose contain of A) hydrogen? B) oxygen? C) carbon? Answer: A) There are 22 hydrogen atoms; B) There are 11 oxygen atoms; C) there are 12 carbon atoms.

Problem 2 - What is the ratio of the number of hydrogen atoms to oxygen atoms? Answer: 22 hydrogen atoms / 11 oxygen atoms so the ratio is 2/1

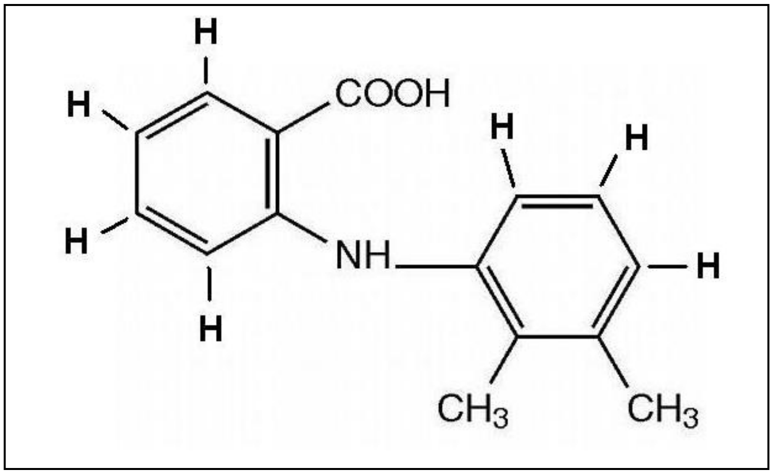
Problem 3 - What fraction of all the maltose atoms are carbon? Answer: The total number of atoms is 12 + 22 + 11 = 45, so carbon atoms are 11/45 of the total.

Problem 4 - If the mass of 1 hydrogen atoms is 1 AMU, and 1 carbon atom is 12 AMU and 1 oxygen atom is 16 AMU, what is the total mass of one maltose molecule in AMUs? Answer: 1 AMU x 22 atoms hydrogen + 12 AMU x 12 atoms carbon + 16 AMU x 11 atoms oxygen = 342 AMU.

Problem 5 - Write the chemical formula of maltose by filling in the missing blanks:

$C_{12}H_{22}O_{11}$

More Challenging Extra Problem:
Below is the structural formula for Mefenamic Acid. Students can determine its chemical formula as $C_{15}H_{15}NO_2$ and its mass as 241 AMU. In this kind of diagram, carbon atoms in the hexagonal rings are located at each vertex. Hydrogen atoms at the vertices are not labeled as well. Challenge your students to GOOGLE the term 'structural diagram' in the 'images' area and try to decipher other more complex molecules such as Lorazepam or Vancocin !



The Murchison meteorite fell to Earth in 1969. This Carbonaceous Chondrite has been extensively studied over the decades, revealing a wealth of complex organic molecules. *(Photo courtesy Chip Clark, Smithsonian Institution)*

The largest known molecules that have been identified in interstellar space, or in meteorites, have not been synthesized with the help of a genetic 'DNA' or 'RNA' blueprint.

In 2005, Drs. Susanna Weaver and Geoffery Blake, both at Caltech, detected the molecule 1,3-Dihydroxyacetone ($C_3H_6O_3$) in the Sagittarius B2 molecular cloud located near the center of the Milky Way galaxy.

Closer to home, in 2010, a detailed study of the famous Murchison meteorite that fell to Earth in 1969, revealed thousands of organic compounds. The largest macromolecular structure was estimated to have a formula $C_{100}H_{70}O_{12}N_3S_2$

In addition, numerous other molecules were found in this meteorite including the protein glycine ($C_2H_5NO_2$), and the sugar ethylene glycol ($C_2H_6O_2$), which is also related to the molecule discovered by Weaver and Blake in the Sagittarius B2 Cloud.

The Murchison meteorite is a member of a carbon-rich family of meteorites that are from a very ancient population of solar system material more than 4.6 billion years old. They contain 20% water by mass, and some of the recovered ones are decidedly 'stinky'.

Of the 6,000,000 kg of meteorites reach Earth's surface each year, fewer than 5% are of this type. When the Earth was still forming, this rate was millions of times higher. During the Late Heavy Bombardment Era between 4.1 and 3.8 billion years ago, after Earth's surface had formed and cooled, a mass delivery rate of 50 million kg/year may have occurred.

Problem 1 – What is the estimated current delivery rate of meteoritic organic molecules to Earth?

Problem 2 – The current terrestrial biomass contains about 4.5×10^{15} kg of complex carbon compounds. At the present rate of deposition, about how long would it take for carbonaceous chondrites to deliver a total mass equal to our present biosphere?

Problem 3 – At the estimated deposition rate of the Late Heavy Bombardment Era, about how long would it take to accumulate Earth's current biosphere mass?

Problem 4 – The total mass of Earth's oceans is about 1.4×10^{21} kg. What percentage of Earth's current ocean mass could have been deposited using Murchison-type Chondrites during the Late Heavy Bombardment Era?

Problem 1 – What is the estimated current delivery rate of meteoritic organic molecules to Earth? Answer: 6,000,000 kg/year x (0.05) = **300,000 kg/year**.

Problem 2 – The current terrestrial biomass contains about 4.5 x 10^{15} kg of complex carbon compounds. At the present rate of deposition, about how long would it take for carbonaceous chondrites to deliver a total mass equal to our present biosphere?
Answer: T = 4.5 x 10^{15} kg / 3 x 10^{5} kg/yr) = **15 billion years**.

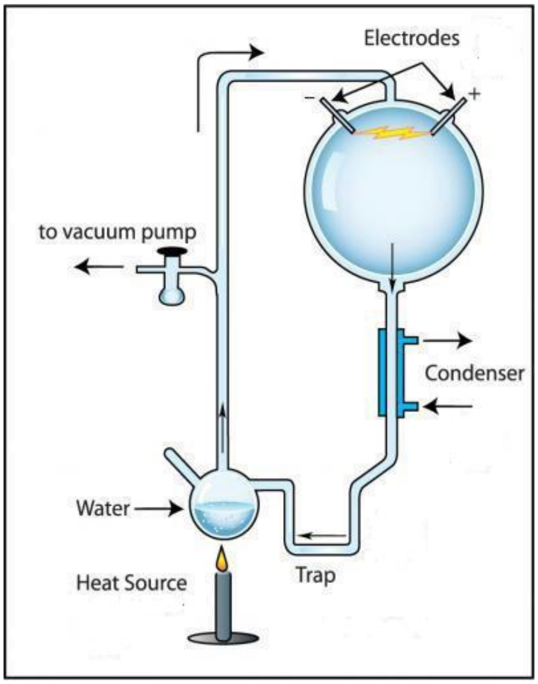
Problem 3 – At the estimated deposition rate of the Late Heavy Bombardment Era, about how long would it take to accumulate Earth's current biosphere mass?

Answer: T = 4.5 x 10^{15} kg / 5 x 10^{7} kg/yr) = **90 million years**.

Problem 4 – The total mass of Earth's oceans is about 1.4 x 10^{21} kg. What percentage of Earth's current ocean mass could have been deposited using Murchison-type Chondrites during the Late Heavy Bombardment Era?

Answer: The chondrites contain 20% by mass of water, so the deposition rate would have been 5 x 10^{7} kg/year x (0.2) = 1 x 10^{7} kg/year. The Late Heavy Bombardment era lasted from 4.1 to 3.8 billion years so the time interval is 300 million years. The total accumulated water would have been about M = 10^{7} kg/yr x 3 x 10^{7} years = 3 x 10^{14} kg. AS a percentage this is about P = 100% x (3 x 10^{14} kg / 1.4 x 10^{21} kg) = **0.00002 %**

This is vastly smaller than the total current mass of Earth's oceans, so only a small portion of Earth's water would have been from this source for the assumed deposition rates and times.



Although some organic 'pre biotic' molecules have been found in the harsh conditions of interstellar space, planetary surfaces provide the best 'factory cite' for creating them in high concentrations and degrees of complexity.

The famous 'Miller-Urey Experiments' conducted in 1953 proved that it was possible to create complex molecules using the ingredients of Earth's primitive atmosphere, and common sources of energy such as lightning (electric sparks) and geothermal heat (Bunsen burner).

The experiment, and other like it, eventually produced dozens of critical amino acids and complex molecules needed to begin the formation of the first replicating molecules (RNA and DNA).

The Miller-Urey Experiment ran for many days, beginning with an atmosphere of methane, ammonia, hydrogen and water - what chemists call a reducing atmosphere (rich in hydrogen). Ten percent of the carbon formed organic compounds such as formaldehyde, and 2% formed amino acids. Among the 40 amino acids were the biologically important glycine, aspartic acid, and alanine, as well as the 'DNA backbone' sugar ribose. One such reaction that produces glycine ($NH_5C_2O_2$) is shown below:

$$NH_2 + CH_2 + CN + \_\_\_H_2O \quad \rightarrow \quad NH_3 + NH_5C_2O_2$$

Problem 1 - Use the principle of Conservation of Mass to 'balance' this reaction by determining how many water molecules are needed so that equal numbers of atoms of each element are present on both sides of the reaction 'arrow'.

Problem 2 - The volume of the original Miller-Urey Experiment was about 0.001 meter^3. Suppose that for every gram of water that was consumed in the reaction, the reaction created 1.64 grams of glycine over a period of 7 days. If this experiment were scaled up to dense clouds in the atmosphere of Earth (volume = 5.0×10^{12} meters^3), how long would it take to 'pollute' Earth's oceans with 1% of the ocean's mass of glycine, if the total mass of the oceans are 1.4×10^{21} kilograms?

Answer Key

Problem 1 - Use the principle of Conservation of Mass to 'balance' this reaction by determining how many water molecules are needed.

Answer: Two are needed; $NH_2 + CH_2 + CN + \mathbf{2}H_2O \rightarrow NH_3 + NH_5C_2O_2$

Note: *The equation says that '2 moles of water yield 1 mole of glycine', provided that the other reactants, NH_2 , CH_2 , and CN are available at the full amounts needed (one mole each). The mass of a water molecule in AMUs is just 2 + 16 = 18 AMUs so 1 mole of water has a mass of 18 grams. For glycine, the mass of this molecule is just 14 + 5 + 24 + 16 = 59 AMUs, so 1 mole of glycine has a mass of 59 grams per mole. So, if we start with 1 gram of water, this is equal to 1 gm x (1 mole / 18 grams) = 0.056 moles of water molecules. The chemical reaction says that we need 2 moles of water to create 1 mole of glycine, so 1 gram of water will create 0.056 / 2 = 0.028 moles of glycine. The mass of glycine will then be 0.028 moles x (59 grams/1 mole) = 1.64 grams.*

Problem 2 - The volume of the original Miller-Urey Experiment was about 0.001 meter^3. Suppose that for every gram of water that was consumed in the reaction, the reaction created 1.64 grams of glycine over a time interval of 7 days. If this experiment were scaled up to dense clouds in the atmosphere of Earth (volume = 5.0×10^{12} meters^3), how long would it take to 'pollute' Earth's oceans with 1% of the ocean's mass of glycine, if the total mass of the oceans are 1.4×10^{21} kilograms?

Answer: The amount of glycine you need is 1% of the ocean mass or 1.4×10^{19} kilograms.

The rate at which the experiment produces glycine is 1.64 grams per week per 0.001 meter^3, or R = 1.64 kilograms/week/meter^3. In more convenient 'annual' units we have 85 kilograms/year/meter^3.

If this reaction took place over the volume of the dense clouds in Earth's atmosphere, the production rate would be 85 kg/yr/ meters^3 x 5.0×10^{12} meters^3 = 4.3×10^{14} kilograms/year.

In order to accumulate the required mass of 1.4×10^{21} kilograms, the 'experiment' would need to run about T = 1.4×10^{21} kilograms / (4.3×10^{14} kilograms/year) = **3 million years**.

DNA and the Genome

A small section of the human Insulin gene:

…TAGGCTGTAGCAGGGGAGCTCTCAAGGGCCCCCCTCTGTGCAG
TTCAGCCGAGGCCAGGCCTGTCCCATCAGGGTGAGGCTTCCCTTG
GGGTGAGGGGTCCTCCTCACCTCCTGATCGTAAAGGCCCAGGGCT
GGGCTAGGGACCCCGGTGTTAAAAGGGCCAGAAAATGCCGTCCAG
GCAGATAGTCCCAGGGAGAGGGCCCAGAGCACCTGCCGCCAGGCT
AGGGACGCACGAGGGGGCCATGGTGTCACCCAGCACCCAGCACCC
CCCAGAAGCTCCTGGATCCCCGTTAAAACTGGGTGATCATTAAGT
GGAGTGCGGTTGTGTCACAGCCCCCAGAAGCCCCACCCACCACTC
CCCAGCCACCCCTGGGAGCAAGGCCCTCACCCAGAGAGGTGGGGT
GTCACCGGCCTGGC…

All living systems on Earth use DNA to transmit information from one generation to the next. This information is in the form of genes that provide information about assembling complex molecules called proteins.

A gene is a sequence of molecules called nucleotides. There are four nucleotide molecules, adenine, guanine, thymine and cytocine, represented by the letters A, G, T and C. A gene can contain thousands of these 'letters' in specific sequences for forming protein molecules where they are 'read out' by the cell's transcription process. Every gene has a 'start' and 'stop' sequence that indicates the boundaries of the gene.

The sequence of letters above is a small part of the gene in humans that provides information for the assembly of a protein called insulin. This gene is found in chromosome 11. It consists of 51 amino acids strung together and re-formed into a 3-dimensional molecule. A process called protein folding. When this gene is active, the cell will excrete insulin molecules into the blood stream. This happens in cell clusters called the 'Islets of Langerhans' located in the pancreas.

Problem 1 - The amino acid Alanine (Ala) is represented by the 3-letter code (GCU, GCC, GCA or GCG) find all of the locations for Alanine in the fragment of the insulin DNA above.

Problem 2 - The amino acid Glycine (Gly) is represented by the 3-letter code is represented by the 3-letter code (GGU, GGC, GGA, or GGG) find all of the locations for Glycine in the fragment of the insulin DNA above.

Problem 4 - What is the ratio of Alanine to Glycine in the fragment of insulin coding?

Problem 3 - Create an imaginary snippet of genetic code where the amino acids Valine (GUC) and Leucine (CUC) are also present with Alanine (GCA) and Glycine (GGC).

A small section of the human Insulin gene:

…TAGGCTGTA**GCA**GGGGAGCTCTCAAGG**GCC**CCCCCTCTGTGCAG
TTCA**GCC**GAG**GCC**AGGCCTGTCCCATCA**GGG**TGA**GGC**TTCCCTTG
GGGTGAG**GGG**TCCTCCTCACCTCCTGATCGTAAA**GGC**CCA**GGG**CT
G**GGC**TAG**GGA**CCCCGGTGTTAAAAG**GGC**CAGAAAATGCCGTCCAG
GCAGATAGTCCCA**GGG**AGA**GGG**CCCAGA**GCA**CCT**GCCGCC**AGGCT
A**GGG**ACGCACGAG**GGG**GCCATGGTGTCACCCAGCACCCAGCACCC
CCCAGAAGCTCCT**GGA**TCCCCGTTAAAACTGGGTGATCATTAAGT
GGAGTGCGGTTGTGTCACA**GCC**CCCAGAAGCCCCACCCACCACTC
CCCA**GCC**ACCCCT**GGG**AGCAAG**GCC**CTCACCCAGAGAGGT**GGG**GT
GTCACCG**GCC**T**GGC**…

Problem 1 - The amino acid Alanine (Ala) is represented by the 3-letter code (GCU, GCC, GCA or GCG) find all of the locations for Alanine in the fragment of the insulin DNA. Answer; See above highlights. There are 9-GCC and 2-GCA

Problem 2 - The amino acid Glycine (Gly) is represented by the 3-letter code is represented by the 3-letter code (GGU, GGC, GGA, or GGG) find all of the locations for Glycine in the fragment of the insulin DNA above. Answer; See above highlights. There are 5-GGC, 2-GGA and 9-GGG

Problem 3 - What is the ratio of Alanine to Glycine in the fragment of insulin coding? Answer: Alanine = 11 and Glycine = 15 so **A:G = 11/15**

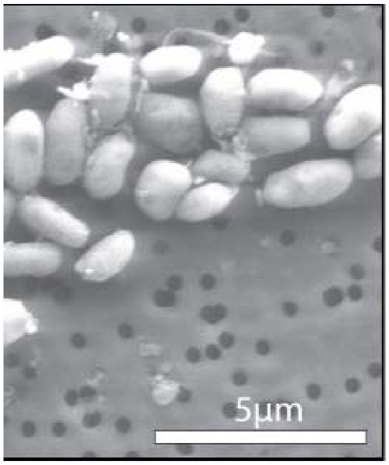
Problem 4 - Create an imaginary snippet of genetic code where the amino acids Valine (GUC) and Leucine (CUC) are also present with Alanine (GCA) and Glycine (GGC). Example:

…CUCGCACUCCUCGUCGGCGGCCUCCUCGUCGGC….

which represents the amino acid sequence

….Leu, Ala, Leu, Leu, Val, Gly, Gly, Leu ,Leu, Val, Gly….

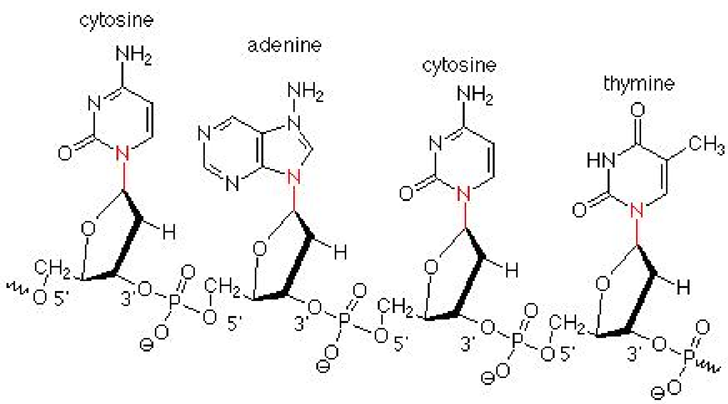
The protein formed from this piece of code would have the above amino acids connected together in that specific order.



Microphotograph of the new bacterium GFAJ-1 that subsists on the toxic element arsenic.

NASA researchers exploring extremophile bacteria in Mono Lake, California recently discovered a new strain of bacterium GFAJ-1 in the Gammaproteobacteria group, which not only feeds on the poisonous element arsenic, but incorporates this element in its DNA as a replacement for normal phosphorus. All other known life forms on Earth use 'standard' DNA chemistry based upon the common elements carbon, oxygen, nitrogen and phosphorus.

In the search for life on other worlds, knowing that 'life' can exist that is fundamentally different than Earth life now broadens the possible places to search for the chemistry of life in the universe.



This diagram shows the elements that make up a small section of normal DNA containing the four bases represented from top to bottom by the sequence 'CACT'. They are held together by a 'phosphate backbone' consisting of a phosphor atom, P, bonded to four oxygen atoms, O. Each phosphor group (called a phosphodiester) links together two sugar molecules (dioxyribose), which in turn bond to each of the bases by a nitrogen atom, N.

Problem 1 - The atomic mass of phosphor P= 31 AMU, arsenic As= 75 AMU, hydrogen H=1 AMU and Oxygen O= 16 AMU. A) What is the total atomic mass of one phosphodiester molecule represented by the formula PO_4 ? B) For the new bacterium, what is the total atomic mass of one arsenate molecule represented by the formula AsO_4?

Problem 2 - The DNA for the smallest known bacterium, mycoplasma genetalium, has about 582,970 base pairs. Suppose that the 1,166,000 phosphodiester molecules contribute about 30% of the total mass of this organism's DNA. If arsenic were substituted for phosphorus to form a twin arsenic-based organism, by how much would the DNA of the new organism increase?

Problem 1 - The atomic mass of phosphor P= 31 AMU, arsenic As= 75 AMU, hydrogen H=1 AMU and Oxygen O= 16 AMU. A) What is the total atomic mass of one phosphodiester molecule represented by the formula PO_4? B) For the new bacterium, what is the total atomic mass of one arsenate molecule represented by the formula AsO_4 ?

Answer: A) PO_4 = 1 Phosphorus + 4 Oxygen
= 1 x 31 AMU + 4 x 16 AMU
= **95 AMU**

B) AsO_4 = 1 Arsenic + 4 Oxygen
= 1 x 75 AMU + 4 x 16 AMU
= **139 AMU**

Problem 2 - The DNA for the smallest known bacterium, mycoplasma genetalium, has about 582,970 base pairs. Suppose that the 1,166,000 phosphodiester molecules contribute about 30% of the total mass of this organism's DNA. If arsenic were substituted for phosphorus to form a twin arsenic-based organism, by how much would the DNA of the new organism increase?

Answer: The arsenic-substituted ester has a mass of 139 AMU compared to the phosphorus-based ester with 95 AMU, so the new molecule AsO_4 is 100% x (95/139) = 68% more massive than PO_4.

Since in the normal DNA the PO_4 contributes 30% of the total DNA mass, the non-PO_4 molecules contribute 70% of the normal mass.

This is added to the new arsenic-based molecule mass for AsO_4 of 30% x1.68 = 50% to get a new mass that is 70% + 50% = **120% heavier** than the original, 'normal' DNA based on PO_4.

So we would predict that the DNA of the twin arsenic-based organism is only 20% more massive than the DNA of the original phosphate-based organism.

Note: Students may have a better sense of the calculation if they start with a concrete amount of 100 grams of normal DNA. Then 70 grams are in the non-PO_4 molecules and 30 grams is in the PO_4 molecules. Because AsO_4 is 68% more massive than PO4, its contribution would be 30 grams x 1.68 = 50 grams. Then adding this to the 70 grams you get 120 grams with is 20 grams more massive than normal DNA for a gain of 120%.

The Evolution of Nucleotides and Genes

The DNA molecule for advanced animals consists of millions of nucleotides that 'code' for specific amino acids in the assembly of large molecules called proteins that are essential for living systems. Each protein is coded by a string of thousands of nucleotides that make up a unit of information called the gene. Tens of thousands of genes are found in DNA molecules, which can be activated individually or in groups to carry out essential functions for living cells.

Since 1990, thousands of organisms have had their 'genes sequenced'. The table below shows the details for a few common organisms.

Nucleotide and Gene Evolution

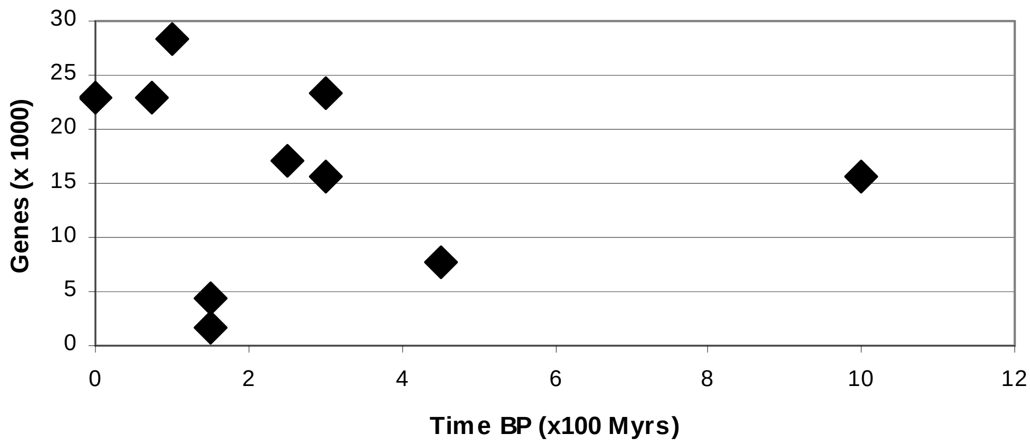
Specie	Type	Age (Myr)	Nucleotides	Genes
Human	Mammal	1	2.9 billion	23,000
Mouse	Mammal	75	3.4 billion	23,000
Rice	Plant	100	390 million	28,300
E.Coli	Bacteria	150	4.6 million	4,400
Zebra Fish	Fish	300	1.2 billion	15,760
Naegleria Gruberi	Ameoba	1,000	41 million	15,727
Streptomyces Coelicolor	Bacteria	450	6.7 million	7,842
Fruit Fly	Insect	250	122 million	17,000
Sea urchin	Mollusk	300	814 million	23,300
Helicobacter Pylori	Bacteria	150	1.7 million	1,589

Problem 1 - Graph the number of genes (in units of thousands) versus time (in units of 100 millions of years). About what is the rate of change of the number of genes per hundred million years to the present time?

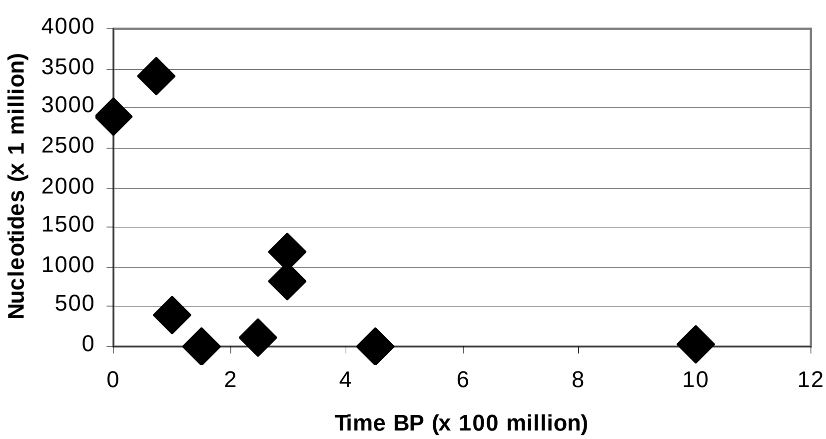
Problem 2 - Graph the number of nucleotides (in millions) versus time (in units of 100 million years). About what is the rate of change in the number of nucleotides per 100 million years to the present time?

Problem 3 - Suppose a planet was discovered that on average had the same kind of biological evolution as Earth during its first 4.6 billion years, but today it is a planet 7 billion years old. About what would you estimate as the number of nucleotides and genes for its most advanced organisms?

Problem 1 - Graph the number of genes (in units of thousands) versus time (in units of 100 millions of years). About what is the rate of change of the number of genes per hundred million years to the present time? Answer: Use the Human and Ameoba data: About G = (23,000 - 15727)/(10) = **700 per 100 million years.**



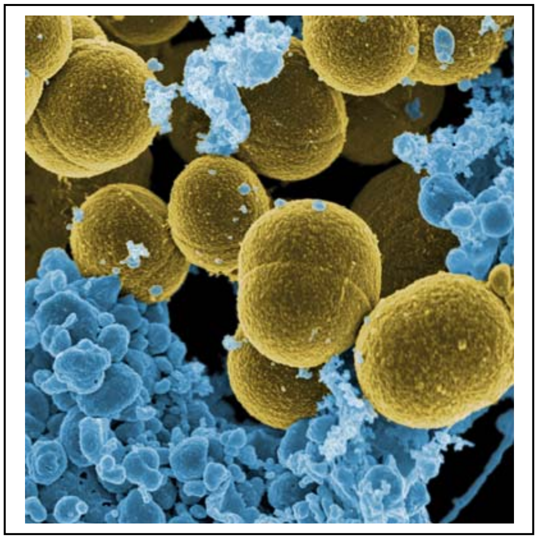
Problem 2 - Graph the number of nucleotides (in millions) versus time (in units of 100 million years). About what is the rate of change in the number of nucleotides per 100 million years to the present time? Answer: Use Mouse and Ameoba data . N = (3.4 billion)/(10-0.75) = **370 million per 100 million years.**



Problem 3 - Suppose a planet was discovered that on average had the same kind of biological evolution as Earth during its first 4.6 billion years, but today it is a planet 7 billion years old. About what would you estimate as the number of nucleotides and genes for its most advanced organisms?

Answer: It is 2400 million years older, so there have been 24, 100million year time intervals more, and at the rates we calculated above we have N = 3.4 billion + 370 million x 24 = **12 billion nucleotides** .Genes = 23,000 + 700 x 24 = **39,800 genes**.

Estimating Maximum Cell Sizes



A simple living cell generates wastes from the volume of cytoplasm inside its cell wall, and passes the wastes out side its wall by passive diffusion.

If a cell cannot remove the waste fast enough, toxins will build up that eventually disrupt cellular functioning. The balance between waste generation and diffusion, therefore, determines how much volume a cell may have and therefore its typical size.

Photo: *S. aureus* bacteria escaping destruction by human white blood cells. (Credit: NIAID / RML)

Suppose the cell has a spherical volume, and that it generates waste at a rate of **A** molecules per cubic micron per second. Suppose it removes the waste through its surface by passive diffusion at a rate of B molecules per square micron per second, where 1 micron is 0.000001 meters.

Problem 1 - What is the equation that defines the rate, R, at which the organism changes the amount of its net waste products?

Problem 2 - For what value for the cell's radius will the net change be zero ,which means the cell is in equilibrium?

Problem 3 - A hypothetical cell metabolism is measured to be a = 800 molecules/μm^3/sec and b = 2000 molecules/μm^2/sec , about how large might such a cell be if it removed waste products only by passive diffusion?

Problem 1 - What is the equation that defines the rate, R, at which the organism changes its net waste products?

Answer:

$$R = \frac{4}{3}\pi r^3 a - 4\pi r^2 b$$

Problem 2 - At what value for the cell's radius will the net change be zero?
Answer: Set R = 0, and then solve for r in terms of *a* and *b* to get:

$$\frac{4}{3}\pi r^3 a = 4\pi r^2 b$$

$$\frac{ra}{3} = b$$

$$r = \frac{3b}{a}$$

Problem 3 - A hypothetical cell metabolism is measured to be a = 800 molecules/μm^3/sec and b = 2000 molecules/μm^2/sec , about how large might such a cell be if it removed waste products only by passive diffusion?

Answer: r = 3 (2000/800) = **7.5 microns in radius.**

Exploring Archaean Life Forms

Methanosarcinae acetavorans has a genome with 5.7 million nucleotides and 4,524 genes. It is highly adapted for producing methane gas as part of its respiration cycle.

Our earliest 'snap shot' of what the first living organism on earth looked like comes from the taxonomic domain 'Archaea', which recently joined 'Bacteria' and 'Eukaryota' as the three major classifications of all living systems on Earth.

Eukaryota (cells with nuclei) contain all familiar plant and animal life on Earth from amoeba to humans, living under relatively temperate conditions of temperature, pressure and salinity. This domain is the most recent, arising perhaps 1 billion years ago.

Bacteria are single-celled, with no nuclei, and which also live under conditions similar to eukaryota, and often for symbiotic relationships with them. This domain extends back in the fossil record to nearly 3.5 billion years ago by some accounts.

Archaea contain a diverse collection of microbes more primitive than bacteria with no cell walls, which can thrive under extreme conditions of temperature, salinity, pressure and radiation. They are believed to be among the earliest forms of life pre-dating the bacteria.

Each year, more than 900 million tons of methane are produced by modern-day organisms in the Archaea domain. Chief among these is *Methanosarcinae acetavorans* found in the anerobic conditions (oxygen-poor) of waste dumps, swamps, garden soil, and the guts of most animals. The microbe takes-in carbon dioxide and acetate and produce methane gas; a respiratory cycle that generates enough metabolic energy to keep them alive.

The respiration pathway involves many steps. Two molecules called *methylcoenzyme-M* and *methylcoenzyme-B* are combined with the help of a catalyst molecule, to create the complex molecule called *heterodisulfide*, with the release of metabolic energy and methane gas. This is similar to the process in humans in which sugars in our food are combined with the oxygen we breathe, and produce carbon dioxide plus metabolic energy. The reaction can be written as follows:

$$\underset{\text{Methyl-CoM}}{C_3H_5O_3S_2} + \underset{\text{Methyl-CoB}}{C_{11}H_{13}O_7NSP} \longrightarrow \underset{\text{Heterodisulfide}}{C_{13}O_{10}H_{14}S_3NP} + \underset{\text{Methane}}{CH_4} + \text{energy}$$

Problem 1 - Show that mass is conserved.

Problem 2 - If 30,000 Joules are generated for every mole of Methyl-CoM consumed, how many liters of methane gas are produced if the microbe colony produces 90,000 Joules of metabolic energy? (Note: 1 mole of gas = 22 liters at room temperature and pressure)

Answer Key

Problem 1 - Show that mass is conserved.

Answer: **Students will count the number of atoms of each element on the left-side of the chemical reaction equation, and see that the same number of atoms are present in the right-side.**

Problem 2 - If 30,000 Joules are generated for every mole of Methyl-CoM consumed, how many liters of methane gas are produced if the microbe colony produces 90,000 Joules of metabolic energy? (Note: 1 mole of gas = 22 liters at room temperature and pressure)

Answer: Because we are given that 30,000 Joules are produced per mole, and that the total energy yield was 90,000 Joules, there must have been 3 moles of Methly-CoM used. We also see in the chemical equation that, for each mole of Methyl-CoM consumed, one mole of methane is generated. So, since 3 moles of methyl-CoB were consumed, there must have been 3 moles of methane gas generated. So the total volume of methane gas generated was 3 moles x (22 liters / 1 mole) = **66 liters** of methane gas at room temperature and pressure.

Note: More information about this microbe is available at the Broad Institute, including a download of the complete nucleotide sequence of 5.7 million bases, and a list iof its known genes including their function:

http://www.broadinstitute.org/annotation/microbes/methanosarcina/background.html

For example, at the link:

http://www.broadinstitute.org/cgi-bin/annotation/methanosarcina/download-sequence.cgi

you can download ***gene_list.txt*** *and have students create a table of the number of genes that perform various functions in the cell, and identify the ones that are responsible for methane respiration cycles, which are identifiable by the prefix 'methyl' in the function name.*

Encephalization is a measure of the size of a mammalian brain compared to its body mass. Although large bodies often require proportionately larger brains to operate all of their systems, encephalization is considered to be a measure of 'excess' brain size, and an indirect measure of a mammalian specie's raw intelligence. The table below gives the 'encephalization quotient' (EQ) for some common mammals along with the age of the species.

Encephalization over Time

Specie	Age (Myr)	EQ	Population Size	Genes
Human	1	7.5	6 billion	24,000
Dolphin	50	4.1	600,000	24,000
Orca	70	2.7	5,000	20,000
Chimpanzee	10	2.3	150,000	19,700
Dog	40	1.2	300 million	19,300
Cat	12	1.0	200 million	20,285
Horse	50	0.9	58 million	20,436
Rat	80	0.4	4 billion	23,000

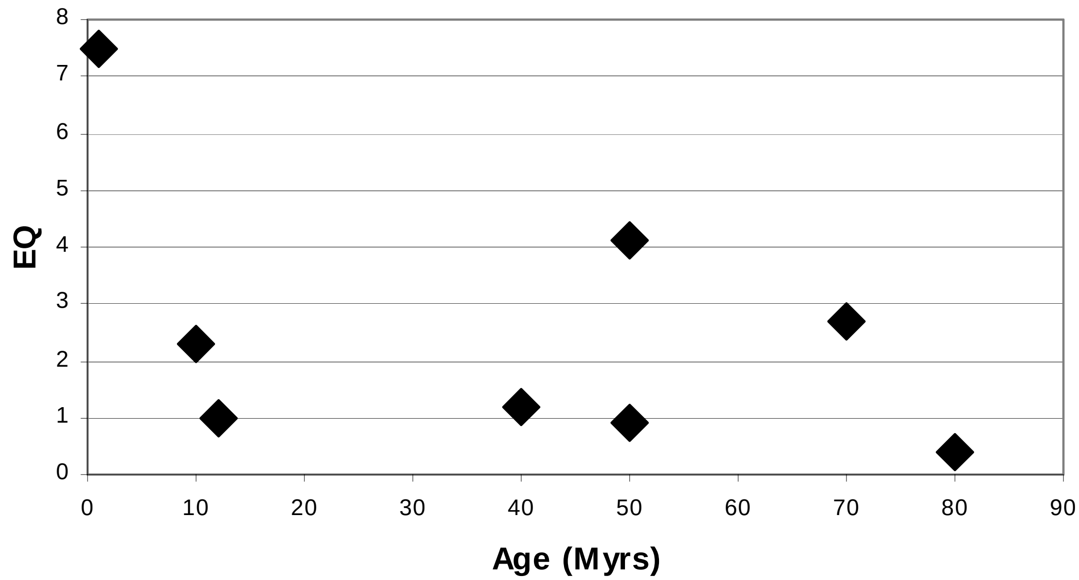
Problem 1 - Graph the EQ versus time (in units of 10 million years). What is the average rate of EQ increase per 10 million years to the present time?

Problem 2 - How many years will it take for the EQ to double?

Problem 3 - Is there any trend between: A) EQ and the number of genes in the species? B) EQ and the size of the species population? C) EQ and the age of the species?

Problem 4 - The average mammalian gene mutation rate has been estimated as 2.2×10^{-9} mutations/per nucleotide/year. The average size of the genome for the above mammal species is about 3 billion, of which 3% is in genes that code for proteins. A) About how many mutations distinguish a rat from a human? B) How many mutations would be in the genetic material?

Problem 1 - Graph the EQ versus time (in units of 10 million years). What is the average rate of EQ increase per 10 million years to the present time? Answer; Using the Human and Rat, the slope is EQ = (7.5 - 0.4)/(79) = **+0.09 EQ points per million years**.



Problem 2 - How many years will it take for the EQ to double? Answer: 1.0 = 0.09x T so T = 1/0.09 or about **10 million years**.

Problem 3 - Is there any trend between: A) EQ and the number of genes in the species? B) EQ and the size of the species population? C) EQ and the age of the species? Answer: No, because there are significant exceptions that disprove the trend, for example A) rats have a comparable gene size to humans but 1/20 the EQ; B) Again, rats and humans have comparable population sizes. C) The next-largest EQ is from the Dolphin which has been around for about 50 million years.

Problem 4 - Answer: A) There are M = 2.2×10^{-9} x 3 billion nucleotides/genome = 6 nucleotide mutations per year per genome. The rat genome is 79 million years older than the human genome, so there are about 79 million x 6 = 474 million possible mutations in the common genome.

B) Since genes occupy about 3% of the nucleotides, there would be about 474 million x 0.03 = 14 million mutations in the genetic material. The rest would occur in the non-coding 'junk DNA', which is involved in gene regulation.

Data from "Mutation Rates in Mammalian Genomes" (2002, S. Kumar; Proc. Nat. Acad. Sci. vol. 99, p. 803)

Gene	Size	Functions
FOX P2	607,463	Speech production and understanding
Beta Catenin	23,200	Brain size
Pax-6	33,170	Eye formation
Otoferlin	101,496	Hearing
RGS-14	14,756	Learning, memory
SIRT-1	33,721	Memory, learning, coping
COMT	28,236	Thinking skills
KRTHAP-1	6,658	Hair growth
Myh-16	24,672	Jaw size and cranium volume
FGF-5	24,430	Hair length
ASPM	62,568	Cerebral cortex growth
MSX-2	6,328	Brain and skull synchrony
FGFR-3	15,461	Brain and skull growth
TWIST	2,205	Brain and skull growth
DTNBP-1	140,258	Problem solving, learning, comprehension

The table to the left presents the genes that have played a role in the development of advanced intelligence on Earth. Mutations in any one of these cause known developmental problems such as microcephaly.

In order for evolution on Earth to have led to advanced intelligence, larger brain sizes and communication skills were a basic requirement, in addition to capabilities for remembering details, learning, and employing advanced thinking skills. All of these attributes were made possible by the activation of specific genes in the mammalian genome.

On an alien world, the assumption is that similar genes affecting the scope and complexity of nervous systems also play a role.

The mammalian genome is about 3 billion nucleotides long. In humans, only 3% is in the form of genes that code for proteins. The total number of mammalian genes is about 24,000 for most species. The significant increase in animal intelligence occurred when Earth had developed a biosphere with tens of millions of animal species.

This dramatic rise in sentience, occurred during the last 50 million years (advanced mammals), and more spectacularly during the last million years (such as Homo Sapiens, Homo Neanderthalensis, Home Heidelbergensis, etc).

Problem 1 - What percentage of mammalian genes are apparently involved in setting the stage for the further development of advanced intelligence?

Problem 2 - The total number of animal species is about 2 million. What percentage achieved advanced intelligence after 600 million years of evolution?

Problem 3 - Explain how inter-species competition may lead to only one surviving species with advanced intelligence living on a planet, at any given time.

Problem 1 - What percentage of mammalian genes are apparently involved in setting the stage for the further development of advanced intelligence?

Answer; P = 100% x (15/24,000) = 0.06 % or one out of 1600.

Problem 2 - The total number of animal species is about 2 million. What percentage achieved advanced intelligence after 600 million years of evolution?

Answer: If we only include Homo Sapiens, it is 1 in 2 million. If we include the three examples in Problem 1, we have 3 in 2 million.

Problem 3 - Explain how inter-species competition may lead to only one surviving species with advanced intelligence living on a planet, at any given time.

Answer: Students may consider many different factors, but should support them with concrete examples and 'extra credit' math elements (quantitative statements or simple calculations). For example, students may consider that species evolution on Earth has been generally violent, with older species replaced through competition with newer species who seek to occupy the same ecological niches.

For advanced intelligences, multiple species can survive if they are isolated, such as dolphins living under aquatic conditions and the genus Homo living on land. However, if the advanced species expands in population size, it will seek to migrate into other ecological niches, and if tool-making, learn how to exist in relatively foreign ecological niches. Students may use the example of the contact between Homo Sapiens and Homo Neanderthalensis, the apparent competition for the same resources (scarce big game and food) and the extinction of the apparently less-adaptable species: Neanderthalensis.

Statistically, although the mutation rate of 1 in 1600 leading to the genetic pre-cursors to advanced intelligence is very high, the actual number of surviving species is very low, suggesting that competition among similarly-endowed species is very intense at each step of the development path to advanced intelligence. Even now, humans barely tolerate the presence of dolphins (the closest advanced intelligence to humans) in the terrestrial ecology, and are in the process of hunting them to extinction because they compete for the same fish species that humans find currently edible on the commercial-scale.

Table of known or suspected extinctions

Event	Time before present	Consequence
Toba Volcanic Eruption	70,000	Human evolution bottleneck
Global Warming	55 million	Paleocene Extinction
Asteroid Impact	65 million	Extinction of dinosaurs
Global Cooling	205 million	Triassic-Jurassic Extinctions
Global Warming	251 million	Permian-Triassic Extinctions
Global Cooling	370 million	Late Devonian Extinctions
Global Cooling	440 million	Ordovician Extinctions
Ocean Anoxia	540 million	End-Ediacaran Extinction
Global Cooling	650 million	Snowball Earth

Scientists have recently returned to the older idea that the evolution of life is often determined by natural cataclysms. These events are random in nature, but pose a major challenge for species to survive them. Many do not, as we can see from the many 'Mass Extinctions that have occurred. One may rightly wonder how other planetary systems may have had multiple episodes of life originating, but some cataclysm stopped evolution in its tracks.

We have to remember that, on Earth, over 99% of all species that have ever existed are extinct.

Scientists mostly agree that 'Extinction-Level Events' (ELEs) require that a large asteroid or comet collide with Earth, and that the sizes have to be at least 5 kilometers in order for there to be enough kinetic energy to severely impact the global biosphere. Earth is comfortably far from the largest concentration of asteroids in the solar system; The Asteroid Belt, however Mars is a potentially habitable planet very close to the Asteroid Belt. It is not hard to imagine that Mars receives far more risk of major impacts that does Earth. There are other natural calamities that can cause extinctions, such as increased episodes of volcanic activity, generating global warming, or the migration of the continents causing climate and ecology changes over millions of years.

Problem 1 - From the table, about what is the average time between major extinction events on Earth?

Problem 2 - A 10-kilometer diameter, spherical asteroid slams into Earth at a speed of V=20,000 m/s. If the asteroid is made of rocky material, its mass could be about M = 2 x 10^{15} kilograms. If its kinetic energy, in Joules, is given by the formula

$$K.E. = \frac{1}{2}MV^2$$

and a typical solar flare produces about 10^{23} Joules of energy, about how large is the impact compared to a typical solar flare on the sun?

Answer Key

Problem 1 - From the table, about what is the average time between major extinction events on Earth?

Answer: Students have to difference the numbers to get the time interval between them:

Event	Time before present	Difference
Toba Volcanic Eruption	70,000	0
Global Warming	55 million	55 million
Asteroid Impact	65 million	10 million
Global Cooling	205 million	140 million
Global Warming	251 million	46 million
Global Cooling	370 million	119 million
Global Cooling	440 million	70 million
Ocean Anoxia	540 million	100 million
Global Cooling	650 million	110 million

The average time interval is just T = (55+10+140+46+119+70+100+110)/8
or **81 million years**.

Problem 2 - A 10-kilometer diameter, spherical asteroid slams into Earth at a speed of V=20,000 m/s. If the asteroid is made of rocky material, its mass could be about M = 2 x 10^{15} kilograms. If its kinetic energy, in Joules, is given by the formula

$$K.E. = \frac{1}{2}MV^2$$

and a typical solar flare produces about 10^{23} Joules of energy, about how large is the impact compared to a typical solar flare on the sun?

Answer:

$K.E. = \frac{1}{2}(2.0x10^{15})(20,000)^2$ so K.E. = **4.0×10^{23} Joules**.

So an Extinction-Level Event corresponds to about as much energy as **four typical solar flares!**

Note: A large solar flare produces about 6 x 10^{25} Joules in an hour or so!

Kelvin Temperatures and Very Cold Things!

183 K	Vostok, Antarctica
160 K	Phobos
134 K	Superconductors
128 K	Europa summer
120 K	Moon at night
95 K	Titan
90 K	Liquid oxygen
88 K	Miranda
81 K	Enceladus summer
77 K	Liquid nitrogen
70 K	Mercury at night
63 K	Solid nitrogen
55 K	Pluto summer
54 K	Solid oxygen
50 K	Quaoar
45 K	Moon - shadowed crater
40 K	Star-forming region
33 K	Pluto wInter
20 K	Liquid hydrogen
19 K	Bose-Einstein Condensates
4 K	Liquid helium
3 K	Cosmic Background Radiation
2 K	Liquid helium
1 K	Boomerang Nebula
0 K	ABSOLUTE ZERO

To keep track of some of the coldest things in the universe, scientists use the Kelvin temperature scale which begins at 0^o Kelvin, which is also called Absolute Zero. Nothing can ever be colder than Absolute Zero because at this temperature, all motion stops. The table to the left shows some typical temperatures of different systems in the universe.

You are probably already familiar with the Centigrade (C) and Fahrenheit (F) temperature scales. The two formulas below show how to switch from degrees-C to degrees-F.

$$C = \frac{5}{9}(F - 32) \qquad F = \frac{9}{5}C + 32$$

Because the Kelvin scale is related to the Centigrade scale, we can also convert from Centigrade to Kelvin (K) using the equation:

$$K = 273 + C$$

Use these three equations to convert between the three temperature scales:

Problem 1: 212 F converted to K

Problem 2: 0 K converted to F

Problem 3: 100 C converted to K

Problem 4: -150 F converted to K

Problem 5: -150 C converted to K

Problem 6: Two scientists measure the daytime temperature of the moon using two different instruments. The first instrument gives a reading of + 107 C while the second instrument gives + 221 F. A) What are the equivalent temperatures on the Kelvin scale; B) What is the average daytime temperature on the Kelvin scale?

Answer Key

$$C = \frac{5}{9}(F - 32) \qquad F = \frac{9}{5}C + 32 \qquad K = 273 + C$$

Problem 1: 212 F converted to K :
First convert to C: C = 5/9 (212 - 32) = +100 C. Then convert from C to K:
K = 273 + 100 = **373 Kelvin**

Problem 2: 0 K converted to F: First convert to Centigrade:
0 = 273 + C so C = -273 degrees. Then convert from C to F:
F = 9/5 (-273) + 32 = **-459 Fahrenheit**.

Problem 3: 100 C converted to K : K = 273 - 100 = **373 Kelvin**.

Problem 4: -150 F converted to K : Convert to Centigrade
C = 5/9 (-150 - 32) = -101 C. Then convert from Centigrade to Kelvin: K = 273 - 101 = **172 Kelvin**.

Problem 5: -150 C converted to K : K = 273 + (-150) = **123 Kelvin**

Problem 6: Two scientists measure the daytime temperature of the moon using two different instruments. The first instrument gives a reading of + 107 C while the second instrument gives + 221 F.

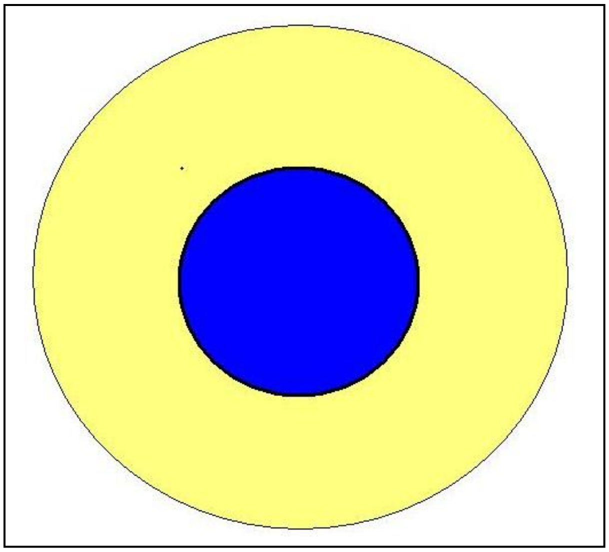
A) What are the equivalent temperatures on the Kelvin scale?;
107 C becomes K = 273 + 107 = **380 Kelvins.**
221 F becomes C = 5/9 (221 - 32) = 105 C, and so K = 273 + 105 = **378 Kelvins**.

B) What is the average daytime temperature on the Kelvin scale?
Answer: (380 + 378)/2 = **379 Kelvins**.

C) Explain why the Kelvin scale is useful for calculating averages of different temperatures. Answer: **Because the degrees are in the same units in the same measuring scale so that the numbers can be averaged.**

Note: Students may recognize that in order to average +107 C and +221 F they could just as easily have converted both temperatures to the Centigrade scale or the Fahrenheit scale and then averaged those temperatures. You may challenge them to do this, and then compare the averaged values in the Centigrade, Fahrenheit and Kelvin scales. They should note that the final answer will be the same as 379 Kelvins converted to F and C scales using the above formulas.

How to Build a Planet from the Inside Out!



The planet Osiris orbits 7 million kilometers from the star HD209458 located 150 light years away in the constellation Pegasus. The Spitzer Space Telescope has recently detected water, methane and carbon dioxide in the atmosphere of this planet. The planet has a mass that is 69% that of Jupiter, and a volume that is 146% greater than that of Jupiter.

By knowing the mass, radius and density of a planet, astronomers can create plausible models of the composition of the planet's interior. Here's how they do it!

Among the first types of planets being detected in orbit around other stars are enormous Jupiter-sized planets, but as our technology improves, astronomers will be discovering more 'super-Earth' planets that are many times larger than Earth, but not nearly as enormous as Jupiter. To determine whether these new worlds are Earth-like, they will be intensely investigated to determine the kinds of compounds in their atmospheres, and their interior structure. Are these super-Earths merely small gas giants like Jupiter, icy worlds like Uranus and Neptune, or are they more similar to rocky planets like Venus, Earth and Mars?

Problem 1 - A hypothetical planet is modeled as a sphere. The interior has a dense rocky core, and on top of this core is a mantle consisting of a thick layer of ice. If the core volume is 4.18×10^{12} cubic kilometers and the shell volume is 2.92×10^{13} cubic kilometers, what is the radius of this planet in kilometers?

Problem 2 - If the volume of Earth is 1.1×10^{12} cubic kilometers, to the nearest whole number, A) how many Earths could fit inside the core of this hypothetical planet? B) How many Earths could fit inside the mantle of this hypothetical planet?

Problem 3 - Suppose the astronomer who discovered this super-Earth was able to determine that the mass of this new planet is 8.3 times the mass of Earth. The mass of Earth is 6.0×10^{24} kilograms. What is A) the mass of this planet in kilograms? B) The average density of the planet in kilograms/cubic meter?

Problem 4 - Due to the planet's distance from its star, the astronomer proposes that the outer layer of the planet is a thick shell of solid ice with a density of 1000 kilograms/cubic meter. What is the average density of the core of the planet?

Problem 5 - The densities of some common ingredients for planets are as follows:

Granite 3,000 kg/m^3 ; Basalt 5,000 kg/m^3 ; Iron 9,000 kg/m^3

From your answer to Problem 4, what is the likely composition of the core of this planet?

Problem 1 - The planet is a sphere whose total volume is given by $V = 4/3\ \pi\ R^3$. The total volume is found by adding the volumes of the core and shell to get $V = 4.18 \times 10^{12} + 2.92 \times 10^{13} = 3.34 \times 10^{13}$ cubic kilometers. Then solving the equation for R we get $R = (3.34 \times 10^{13} / (1.33*3.14))^{1/3} =$ 19,978 kilometers. Since the data is only provided to 3 place accuracy, the final answer can only have three significant figures, and with rounding this equals a radius of R = 20,000 kilometers.

Problem 2 - If the volume of Earth is 1.1×10^{12} cubic kilometers, to the nearest whole number,
A) how many Earths could fit inside the core of this hypothetical planet?
Answer: $V = 4.18 \times 10^{12}$ cubic kilometers / 1.1×10^{12} cubic kilometers = 4 Earths.

B) How many Earths could fit inside the mantle of this hypothetical planet?
Answer: $V = 2.92 \times 10^{13}$ cubic kilometers / 1.1×10^{12} cubic kilometers = 27 Earths.

Problem 3 - What is A) the mass of this planet in kilograms? Answer: $8.3 \times 6.0 \times 10^{24}$ kilograms = 5.0×10^{25} kilograms.
B) The average density of the planet in kilograms/cubic meter?
Answer: Density = total mass/ total volume
= 5.0×10^{25} kilograms/3.34×10^{13} cubic kilometers
= 1.5×10^{12} kilograms/cubic kilometers.
Since 1 cubic kilometer = 10^9 cubic meters,
= 1.5×10^{12} kilograms/cubic kilometers x (1 cubic km/10^9 cubic meters)
= 1,500 kilograms/cubic meter.

Problem 4 - We have to subtract the total mass of the ice shell from the mass of the planet to get the mass of the core, then divide this by the volume of the core to get its density. Mass = Density x Volume, so the shell mass is $1{,}000\ kg/m^3 \times 2.92 \times 10^{13}\ km^3 \times (10^9\ m^3/km^3) = 2.9 \times 10^{25}$ kg. Then the core mass = 5.0×10^{25} kilograms - 2.9×10^{25} kg = 2.1×10^{25} kg. The core volume is $4.18 \times 10^{12}\ km^3 \times (10^9\ m^3/km^3) = 4.2 \times 10^{21}\ m^3$, so the density is $D = 2.1 \times 10^{25}\ kg / 4.2 \times 10^{21}\ m^3 = 5{,}000\ kg/m^3$.

Problem 5 - The densities of some common ingredients for planets are as follows:
Granite 3,000 kg/m^3 ; Basalt 5,000 kg/m^3 ; Iron 9,000 kg/m^3
From your answer to Problem 4, what is the likely composition of the core of this planet?
Answer: Basalt.

Note that, although the average density of the planet (1,500 kg/m^3) is not much more than solid ice (1,000 kg/m^3), the planet has a sizable rocky core of higher density material. Once astronomers determine the size and mass of a planet, these kinds of 'shell-core' models can give valuable insight to the composition of the interiors of planets that cannot even be directly imaged and resolved! In more sophisticated models, the interior chemistry is linked to the temperature and location of the planet around its star, and proper account is made for the changes in density inside a planet due to compression and heating. The surface temperature, which can be measured from Earth by measuring the infrared 'heat' from the planet, also tells which compounds such as water can be in liquid form.

A very common way to describe the atmosphere of a planet is by its 'scale height'. This quantity represents the vertical distance above the surface at which the density or pressure of the atmosphere decreases by exactly 1/e or $(2.718)^{-1}$ times (equal to 0.368).

The scale height, usually represented by the variable **H**, depends on the strength of the planet's gravity field, the temperature of the gases in the atmosphere, and the masses of the individual atoms in the atmosphere. The equation to the left shows how all of these factors are related in a simple atmosphere model for the density P. The variables are:

$$P(z) = P_0 \, e^{-\frac{z}{H}} \quad \text{and} \quad H = \frac{kT}{mg}$$

z: Vertical altitude in meters
T: Temperature in Kelvin degrees
m: Average mass of atoms in kilograms
g: Acceleration of gravity in meters/sec^2
k: Boltzmann's Constant 1.38×10^{-23} J/deg

Problem 1 - For Earth, g = 9.81 meters/sec^2, T = 290 K. The atmosphere consists of 22% O_2 (m= $2 \times 2.67 \times 10^{-26}$ kg) and 78% N_2 (m= $2 \times 2.3 \times 10^{-26}$ kg). What is the scale height, H?

Problem 2 - Mars has an atmosphere of nearly 100% CO_2 (m=7.3×10^{-26} kg) at a temperature of about 210 Kelvins. What is the scale height H if g= 3.7 meters/sec^2?

Problem 3 - The Moon has an atmosphere of nearly 100% sodium (m=6.6×10^{-26} kg). If the scale height deduced from satellite observations is 120 kilometers, what is the temperature of the atmosphere if g= 1.6 meters/sec^2?

Problem 4 - At what altitude on Earth would the density of the atmosphere P(z) be only 10% what it is at sea level, P_0?

Problem 1 - Answer: First we have to calculate the average atomic mass.

$<m> = 0.22\ (2 \times 2.67 \times 10^{-26}\ \mathrm{kg}) + 0.78\ (2 \times 2.3 \times 10^{-26}\ \mathrm{kg}) = 4.76 \times 10^{-26}\ \mathrm{kg}$. Then,

$$H = \frac{(1.38 \times 10^{-23})(290)}{(4.76 \times 10^{-26})(9.81)} = $$ **8,570 meters or about 8.6 kilometers.**

Problem 2 - Answer:

$$H = \frac{(1.38 \times 10^{-23})(210)}{(7.3 \times 10^{-26})(3.7)} = $$ **10,700 meters or about 10.7 kilometers.**

Problem 3 - Answer

$$T = \frac{(6.6 \times 10^{-26})(1.6)(120000)}{(1.38 \times 10^{-23})} = $$ **918 Kelvins.**

Problem 4 - Answer: $0.1 = e^{-(z/H)}$, Take ln of both sides, $\ln(0.1) = -z/H$ then $z = 2.3\ H$ so for H= 8.6 km, **z = 19.8 kilometers**.

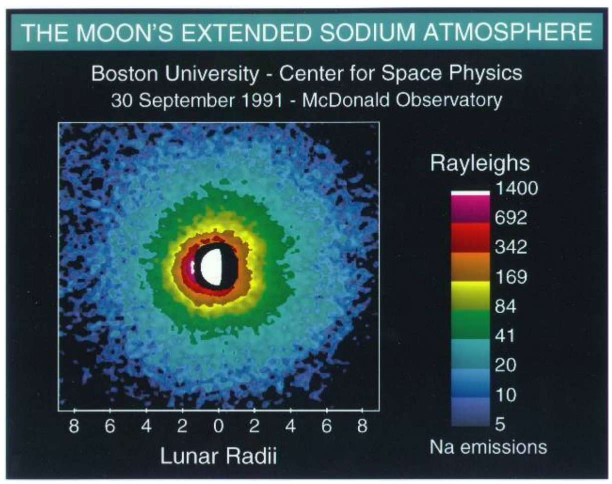
The Moon's Atmosphere!


Courtesy: T.A.Rector, I.P.Dell'Antonio (NOAO/AURA/NSF)

Experiments performed by Apollo astronauts were able to confirm that the moon does have a very thin atmosphere.

The Moon has an atmosphere, but it is very tenuous. Gases in the lunar atmosphere are easily lost to space. Because of the Moon's low gravity, light atoms such as helium receive sufficient energy from solar heating that they escape in just a few hours. Heavier atoms take longer to escape, but are ultimately ionized by the Sun's ultraviolet radiation, after which they are carried away from the Moon by solar wind.

Because of the rate at which atoms escape from the lunar atmosphere, there must be a continuous source of particles to maintain even a tenuous atmosphere. Sources for the lunar atmosphere include the capture of particles from solar wind and the material released from the impact of comets and meteorites. For some atoms, particularly helium and argon, outgassing from the Moon's interior may also be a source.



Problem 1: The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Problem 2: The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU = 1.6×10^{-24} grams, a) How many grams of hydrogen are in one cm3 of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Problem 3: Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Problem 4. Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms? C) metric tons?

Answer Key:

Problem 1: The Cold Cathode Ion Gauge instrument used by Apollo 12, 14 and 15 recorded a daytime atmosphere density of 160,000 atoms/cc of hydrogen, helium, neon and argon in equal proportions. What was the density of helium in particles/cc?

Answer: Each element contributes 1/4 of the total particles so hydrogen = 40,000 particles/cc; helium = 40,000 particles/cc, argon=40,000 particles/cc and argon=40,000 particles/cc

Problem 2: The atomic masses of hydrogen, helium, neon and argon are 1.0 AMU, 4.0 AMU, 20 AMU and 36 AMU. If one AMU = 1.6 x 10-24 grams, a) How many grams of hydrogen are in one cm3 of the moon's atmosphere? B) Helium? C) Neon? D) Argon? E) Total grams from all atoms?

Answer: A) Hydrogen = 1.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 6.4 x 10^{-20} grams
B) Helium = 4.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 2.6 x 10^{-19} grams
C) Neon = 20.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 1.3 x 10^{-18} grams
D) Argon = 36.0 x (1.6 x 10^{-24} grams) x 40,000 particles = 2.3 x 10^{-18} grams
E) Total = (0.064 + 0.26 + 1.3 + 2.3) x 10^{-18} grams = 3.9 x 10^{-18} grams per cc.

Problem 3: Assume that the atmosphere fills a spherical shell with a radius of 1,738 kilometers, and a thickness of 170 kilometers. What is the volume of this spherical shell in cubic centimeters?

Answer: Compute the difference in volume between A sphere with a radius of Ri = 1,738 km and Ro = 1,738+170 = 1,908 km. V = 4/3 π $(1908)^3$ - 4/3π $(1738)^3$ = 2.909 x 10^{10} km^3 - 2.198 x 10^{10} km^3 = 7.1 x 10^9 km^3

Volume = 7.1 x 10^9 km^3 **x** (10^5 cm/km) **x** (10^5 cm/km) **x** (10^5 cm/km)
= 7.1 x 10^{24} cm^3

Note: If you use the 'calculus technique' of approximating the volume as the surface area of the shell with a radius of Ri, multiplied by the shell thickness of h = 170 km, you will get a slightly different answer of 6.5 x 10^9 km^3 or 6.5 x 10^{24} cm^3

Problem 4. Your answer to Problem 2E is the total density of the lunar atmosphere in grams/cc. If the atmosphere occupies the shell whose volume is given in Problem 3, what is the total mass of the atmosphere in A) grams? B) kilograms?

A) Mass = density x volume = (3.9 x 10^{-18} gm/cc) x 7.1 x 10^{24} cm^3 = 2.8 x 10^7 grams

B) Mass = 2.8 x 10^7 grams x (1 kg/1000 gms) = 28,000 kilograms.

C) Mass = 28,000 kg x (1 ton / 1000 kg) = 28 tons.

Teacher note: You may want to compare this mass to some other familiar objects. Also, the Apollo 11 landing and take-off rockets ejected about 1 ton of exhaust gases. Have the students discuss the human impact (air pollution!) on the lunar atmosphere from landings and launches.

How Hot is that Planet?

Planets have been spotted orbiting hundreds of nearby stars, but this makes for a variety of temperatures depending on how far the planet is from its star and the stars luminosity.

The temperature of the planet will be about

$$T=273\left(\frac{(1\text{-}A)L}{D^2}\right)^{1/4}$$

where A is the reflectivity (albedo) of the planet, L is the luminosity of its star in multiples of the sun's power, and D is the distance between the planet and the star in Astronomical Units (AU). The resulting temperature will be in units of Kelvin degrees. (i.e. 0 Centigrade = +273 K, and Absolute Zero is defined as 0 K)

Problem 1 - Earth is located 1.0 AU from the sun, for which L = 1.0. What is the surface temperature of Earth if its albedo is 0.4?

Problem 2 - At what distance would Earth have the same temperature as in Problem 1 if the luminosity of the star were increased 1000 times and all other quantities remained the same?

Problem 3 - The recently discovered planet CoRoT-7b (see artist's impression above, from ESA press release), orbits the star CoRoT-7 which is a sun-like star located about 490 light years from Earth in the direction of the constellation Monoceros. If the luminosity of the star is 70% of the sun's luminosity (L = 0.71) and the planet is located 2.6 million kilometers from its star (D= 0.017 AU) what are the predicted surface temperatures of the day-side of CoRoT-7b for the range of albedos shown in the table below?

Surface Material	Example	Albedo (A)	Surface Temperature (K)
Basalt	Moon	0.06	
Iron Oxide	Mars	0.16	
Water+Land	Earth	0.40	
Gas	Jupiter	0.70	

Problem 1 - Earth is located 1.0 AU from the sun, for which L = 1.0. What is the surface temperature of Earth if its albedo is 0.4? **Answer:** $\mathbf{T = 273\ (0.6)^{1/4} = 240\ K}$

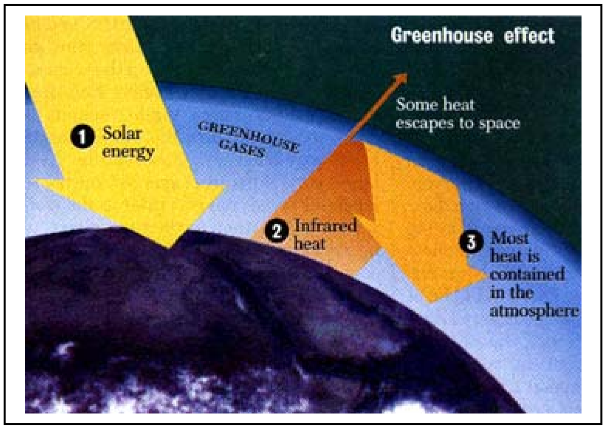
Problem 2 - At what distance would Earth have the same temperature as in Problem 1 if the luminosity of the star were increased 1000 times and all other quantities remained the same? Answer: From the formula, T = 240 and L = 1000 so
$240 = 273(0.6 \times 1000/D^2)^{1/4}$ and so **D = 5.6 AU**. This is about near the orbit of Jupiter.

Problem 3 - The recently discovered planet CoRoT-7b orbits the star CoRoT-7 which is a sun-like star located about 490 light years from Earth in the direction of the constellation Monoceros. If the luminosity of the star is 71% of the sun's luminosity (L = 0.71) and the planet is located 2.6 million kilometers from its star (D= 0.017 AU) what are the predicted surface temperatures of the day-side of CoRoT-7b for the range of albedos shown in the table below?

Surface Material	Example	Albedo (A)	Surface Temperature (K)
Basalt	Moon	0.06	1892
Iron Oxide	Mars	0.16	1840
Water+Land	Earth	0.40	1699
Gas	Jupiter	0.70	1422

Example: For an albedo similar to that of our Moon:
$T = 273 * ((1\text{-}0.06)*0.71/(0.017)^2)^{.25}$
= **1,892 Kelvin**

Note: To demonstrate the concept of Significant Figures, the values for L, D and A are given to 2 significant figures, so the answers should be rounded to 1900, 1800, 1700 and 1400 respectively



An important property of some planets, and a very desirable one for the origin of life, is the presence of an atmosphere!

A bare planet will reach a much lower equilibrium temperature if it does not have an atmosphere to serve as an insulating layer to trap some of the out-going energy that the planet is emitting.

The maximum equilibrium temperature for a planet with an albedo of A, with no atmosphere, and at a distance of D astronomical units from the sun, the surface temperature is given by $T = [6.0 \times 10^9 (1 - A) / D^2]^{1/4}$ and so for Earth with D = 1.0 AU, and in this case A = 0.35, we get T = 249 K. This is -24 C and so water would be permanently in ice form. But we know that the global earth temperature is closer to +14 C ad we have plenty of liquid water! How do we add-in the effect of Greenhouse heating caused by carbon dioxide to increase planetary temperatures?

The following formula, based on terrestrial data between 1960 and 2000, predicts the average global temperature for various atmospheric concentrations, C, of carbon dioxide, expressed in 'parts per million' (expressed as the unit ppm). In 1960, the concentration was 315 ppm, but it has since increased to 385 ppm by 2005.

$$\mathbf{T = 249\,[\,1 + 0.77\,(C/315)^{1/8}\,]^{1/4}}$$

For C = 315 ppm: $T = 249 \times [\,1 + 0.77]^{1/4}$
$= 287$ K or +14 C

For C = 385 ppm: $T = 249\,[1 + 0.77\,(385/315)^{1/8}\,]^{1/4}$
$= 249 \times [1 + 0.77 \times (1.22)^{1/8}\,]^{1/4}$
$= 249 \times [1.79]^{1/4}$
$= 288$ K or +15 C

So the growth in atmospheric carbon dioxide has increased the global temperature by about 1 C since 1960. (The actual data indicates T(1960) = +14C and T(2005) = so the change has been about +14.5 C.)

Problem 1 - The planet Mars is located 1.52 AU from the sun and instead of a minimum temperature of 249 K its temperature is 202 K. What would the carbon dioxide concentration have to be so that the surface temperature remained at 1 C (274 K)? [Hint: Solve the formula exactly for C, or program the formula for T into an Excel Spreadsheet, and make successive guessed for C until you get close to the desired temperature.]

Problem 1 - The planet Mars is located 1.52 AU from the sun and instead of a minimum temperature of 249 K its temperature is 202 K. What would the carbon dioxide concentration have to be so that the surface temperature remained at 1 C (274 K)? [Hint: Solve the formula exactly for C, or program the formula for T into an Excel Spreadsheet, and make successive guessed for C until you get close to the desired temperature.]

$T = 202 [1 + 0.77 (C/315)^{1/8}]^{1/4}$.

To solve this problem exactly, you want T = 274 so

$274 = 202 [1 + 0.77 (C/315)^{1/8}]^{1/4}$

$274/202 = [1 + 0.77 (C/315)^{1/8}]^{1/4}$

$1.36 = [1 + 0.77 (C/315)^{1/8}]^{1/4}$

$3.42 = 1 + 0.77 (C/315)^{1/8}$: Raise both side to the power of 4 to clear the '1/4'

$2.42 = 0.77 (C/315)^{1/8}$

$3.14 = (C/315)^{1/8}$: Divide both side by 0.77

$9450 = C/315$: Raise both side to the power of 8

1,086,750 parts per million !

'One million parts per million' is another way of saying that the atmosphere has to be 100% carbon dioxide and similar to Earth in all other aspects…particularly its density!

Some scientists think that Mars can be 'terraformed' by vaporizing its polar ice caps, which would make the atmosphere nearly as dense as earths, and 100% carbon dioxide. This would raise the surface temperature high enough for liquid water to exist.

Studies of ancient Mars geology and landforms also suggest that it had an atmosphere like this several billion years ago, which is why there are so many features on its surface that indicate that running water must have existed on Mars long ago.

Albedo and Heat Balance

Albedos for common objects

Material	Albedo
Asphalt	0.04
Moon surface	0.12
Mars surface	0.15
Bare Soil	0.17
Green Grass	0.25
Solid Ice	0.35
Desert Sand	0.40
Concrete	0.50
Venus	0.65
Fresh Snow	0.95

Snow reflects so much sunlight that skiers need sunglasses in order to ski comfortably. Meanwhile, asphalt hardly reflects any light at all, which is why it appears so dark! The technical term for this basic property of matter is its **albedo,** represented by the capital letter A. The albedo of bodies varies from 0, which means they reflect absolutely NO light, up to 1.0, which means that they reflect 100% of ALL the light energy that falls on them. A mirror has an albedo of more than 0.99 while the darkest known black color has an albedo of 0.0001.

Albedo is important in the heating and cooling of bodies because it determine just how much of the solar energy falling on a body will be absorbed by it. The amount that is absorbed is exactly equal to 1 - A. If a body has an albedo of 0.8, that means it will absorb 20% of the light energy falling on it, and reflect the remaining 80%.

At the distance of Earth from the Sun, sunlight has an energy flux of 1,357 Watts/meter^2. Use this information, and the table, to answer the following questions. Assume that all materials are 1 square meter in size.

Problem 1 - For asphalt on a sunny day; A) How many watts of heat energy is absorbed? B) How many watts are emitted?

Problem 2 - For solid ice; A) How many watts of heat energy is absorbed? B) How many watts are emitted?

Problem 3 - At Mars, the solar energy flux is 587 watts/meter^2 and at Venus the solar energy flux is 2,618 watts/meter^2. How much more energy does a 1 square meter area of bare soil absorb at Venus that at Mars?

Problem 4 - At Mars, a 1 kilogram block of ice with an albedo of 0.35, has a surface area of 0.06 meters^2. At 0 C it emits P(out)=21 watts of heat energy. It requires 330,000 Joules of energy to be absorbed in order to fully melt. Will such a block of water ice at Mars melt?

Problem 1 - For asphalt on a sunny day; A) How many watts of heat energy is absorbed? B) How many watts are emitted?

Answer; A) The albedo of asphalt is 0.04, so the amount of solar energy reflected is 0.04 x 1,357 watts/meter2 x 1 meter^2 = 54 watts. B) The amount absorbed is (1 - 0.04) x 1,357 watts/meter2 x 1 meter^2 = 1,303 watts.

Problem 2 - For solid ice; A) How many watts of heat energy is absorbed? B) How many watts are emitted?

Answer; A) The albedo of ice is 0.35, so the amount of solar energy reflected is 0.35 x 1,357 watts/meter^2 x 1 meter^2 = 475 watts. B) The amount absorbed is (1 - 0.35) x 1,357 watts/meter^2 x 1 meter^2 = 882 watts.

Problem 3 - At Mars, the solar energy flux is 587 watts/meter^2 and at Venus the solar energy flux is 2,618 watts/meter^2. How much more energy does a 1 square meter area of bare soil absorb at Venus that at Mars?

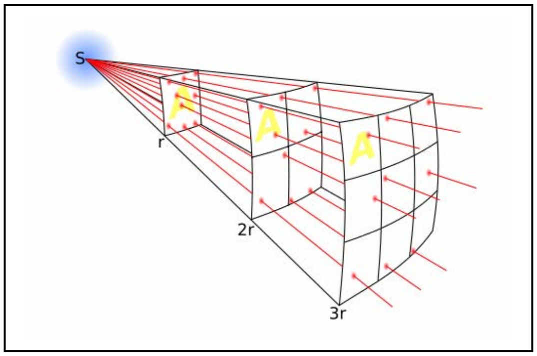
Answer: The albedo of bare soil is 0.17, so at Venus, the amount of energy absorbed is (1 - 0.17) x 2618 watts/meter^2 x 1 meter^2 = 2,173 Watts, while at Mars it is (1 - 0.17) x 587 watts/meter^2 x 1 meter^2 = 487 Watts. The difference is that at Venus the same surface will absorb 2,173 watts - 487 watts = 1,686 watts more solar energy.

Problem 4 - At Mars, a 1 kilogram block of ice with an albedo of 0.35, has a surface area of 0.06 meters^2. At 0 C it emits 20 watts of heat energy. It requires 330,000 Joules of energy to be absorbed in order to fully melt. Will such a block of water ice at mars melt?

Answer: At Mars, from the information in Problem 3, the solar radiation is 587 watts/meter^2. The surface area of the ice is 0.06 meters^2, so the amount of solar energy reaching the ice is 35 watts.

With an albedo of 0.35, ice will absorb (1-.35) x 35 watts = 23 watts.

The ice is originally at 0 C and radiates P(out) = 21 watts of heat energy, and it receives P(in)= 23 watts of energy from the solar radiation, so the net energy gain P = P(in) - P(out) will be P = 3 watts. Because this difference is positive, the ice will accumulate more heat than it radiates away, so it will eventually melt. If P were exactly zero, the ice would not melt.



In the figure above, S is the emitted power of the radiation source. The flux of this energy through space is represented by the red rays passing through the gridded surface areas, which are actually portions of spherical surfaces at radii of r, 2r and 3r. Notice how the density of the rays (called the 'flux') decreases with distance from the light source at S.

When you stand close to the stove you feel warmer than when you stand across the room. This is an example of the 'Inverse-square' law for radiation. There are two parts to the Inverse-square law.

The first part is the total amount of radiant energy involved. A 10-watt bulb feels a lot cooler and looks dimmer, than a 100-watt bulb at the same distance. This is called the **Luminosity** of the source.

The second part is how much of this luminous power passes through the surface area of a sphere centered on the source at a specific distance. This is called the **Flux** of the radiant energy and is related to the intensity of the light.

Example: A radio transmitter on Earth sends out 50,000 watts of power into space in all directions. The Moon is located 384,000 kilometers from Earth. The radius of the sphere is then 384,000 kilometers or 384,000 meters. The surface area of the sphere centered on Earth with this radius is 4 x π x $(384{,}000{,}000)^2$ = 1.8 x 10^{18} meters^2. The flux of radio energy at the Moon is then 50,000 watts/1.8 x 10^{18} meters^2 = 2.8 x 10^{-14} watts/meter^2.

Problem 1 - Using the example as a guide, complete the table below to determine the flux of solar energy at the distance of each of the indicated bodies. The total solar power is 3.8 x 10^{26} watts. Calculate the solar power, in watts, for a solar energy collector with an area of 15 meters^2 at the indicated planetary distances. Report all answers to 2 significant figures.

Object	Distance (AU)	Surface Area (meter^2)	Flux (Watts/m^2)	Power (Watts)
Mercury	0.39			
Venus	0.72			
Earth	1.00			
Mars	1.52			
Saturn	9.54			
Pluto	39.44			

Note: 1 Astronomical Unit (AU) = 149 million kilometers.

Answer Key

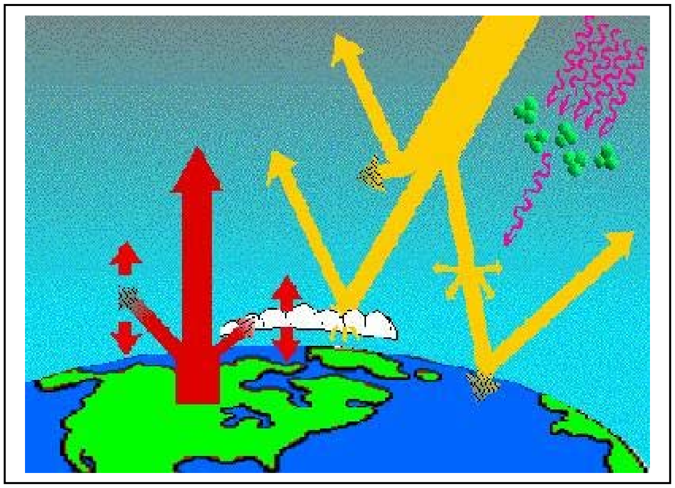
Object	Distance (AU)	Surface Area (meter^2)	Flux (Watts/m^2)	Power (Watts)
Mercury	0.39	4.3×10^{22}	8,800	130,000
Venus	0.72	1.2×10^{23}	3,200	48,000
Earth	1.00	2.8×10^{23}	1,400	21,000
Mars	1.52	6.5×10^{23}	580	8,700
Saturn	9.54	2.5×10^{25}	15	230
Pluto	39.44	4.4×10^{26}	1	15

Colume 3:
Surface Area $= 4 \pi (1.5 \times 10^{11})^2 = 2.8 \times 10^{23}$ m^2 at 1 AU.
So by re-scaling the surface area at 1.0 AU, at 0.39 AU, the area = $(0.39)^2 \times 2.8 \times 10^{23} = 4.3 \times 10^{22}$ m^2

Colume 4:
The Flux will be the solar power, 3.8×10^{26} Watts divided by the surface area.
Example, for Mercury Flux = 3.8×10^{26} Watts/4.3×10^{22} meters^2 = 8,800 watts/m^2.

Colume 5:
At Mars, an area of 15 meters^2 will collect 580 watts/meter^2 x 15 meters^2 = 8,700 watts of solar power.

Heat Flow Balance and Melting Ice



This figure shows solar radiation being absorbed by Earth to heat it (yellow arrows) and being radiated back into space to cool it (red arrows). The difference between these heating and cooling energy flows determines the temperature of Earth, and in a similar way, whether ice will melt. (Courtesy NOAA)

Whether a block of ice melts or not depends on how much energy it is absorbing from its environment, and how much it is radiating away to cool itself. When it absorbs more than it loses, the steady accumulation of energy eventually causes the ice to melt.

The 'magic number' is 330,000 Joules for each kilogram of ice at 0 C. If the ice accumulates more than this energy, it will melt. If it accumulates less than this energy, it remains frozen.

If a 1 kilogram block of ice is left out in the sunlight on Earth, solar energy baths the block of ice with a flux of F(in) = 1,357 Watts/meter^2 of energy. At the same time, this ice at 0 C will emit a flux of energy equal to F(out) = 320 Watts/meter^2. The rate at which the ice is accumulating energy is just F(ice) = F(in) - F(out) which equals 1,357 - 320 = **1,037 Watts/meter^2**. When F(ice) x Time = 330,000 Joules, the ice will melt. Let's look at this process step-by-step.

Problem 1 - The density of ice is 917 kilograms/meter^3. How many cubic meters does 1 kilogram of ice occupy?

Problem 2 - What are the dimensions, in meters, of a perfect cube with the volume found in Problem 1?

Problem 3 - What is the surface area of a cube with the dimensions you found in Problem 2?

Problem 4 - From the value of F(in), how many watts of solar energy, Pi, will the cube of ice absorb if it is illuminated on all sides?

Problem 5 - From the value of F(out), how many watts of energy, Po, is the cube of ice emitting in order to keep cool?

Problem 6 - In units of hours to two significant figures, how long will it take the ice to melt?

Problem 7 - Near Jupiter, F(in) = 53 watts/m^2. How long will the ice take to melt?

Problem 1 - The density of ice is 917 kilograms/meter^3. How many cubic meters does 1 kilogram of ice occupy?

Answer: Volume = Mass/Density so

Volume = 1 kg / 917 kg/meter^3

= 0.001 meter^3

Problem 2 - What are the dimensions, in meters, of a perfect cube with the volume found in Problem 1?

Answer: A cube with dimensions of 0.1 meter x 0.1 meter x 0.1 meter = 0.001 meter^3.

Problem 3 - What is the surface area of a cube with the dimensions you found in Problem 2?

Answer: One face has an area of 0.1 meter x 0.1 meter = 0.01 meter^2, and for 6 faces you get a total surface area of 0.06 meter^2.

Problem 4 - From the value of F(in), how many watts of solar energy, Pi, will the cube of ice absorb if it is illuminated on all sides?

Answer: F(in) = 1,357 Watts/meter^2 so since the total cube surface area is 0.06 meter^2 the total absorbed power will be Pi = 1,357 x 0.06 = 81 watts.

Problem 5 - From the value of F(out), how many watts of energy, Po, is the cube of ice emitting in order to keep cool?

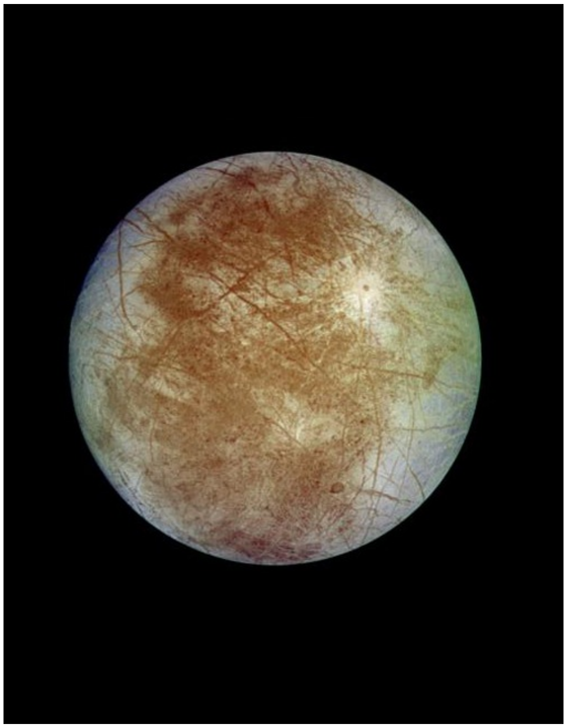
Answer: F(out) = 320 Watts/meter^2 and for the computed cube surface area you will get Po = 320 x 0.06 = 19 watts.

Problem 6 - In units of hours to two significant figures, how long will it take the ice to melt?

Answer: The cube is absorbing a net power of Pi - Po = (81 watts - 19 watts) = 62 watts. In order to melt, it needs to accumulate 330,000 Joules, to the time required will be about T = 330,000 Joules/62 watts = 5,322 seconds or 1.5 hours.

Problem 7 - Near Jupiter, F(in) = 53 watts/m^2. How long will the ice take to melt?
Answer: P(in) = 53 x 0.06 = 3 watts, then Pi - Po = 3 - 19 = -16 watts. Because the heat energy into the ice is <u>less</u> that what the ice is emitting into space, the ice will stay frozen and not accumulate any net heat.

Ice or Water?



There are no known extremophiles that can exist at a temperature lower than the freezing temperature of water. It is believed that liquid water is a crucial ingredient to the chemistry that leads to the origin of life. To change water-ice to liquid water requires energy.

First, you need energy to raise the ice from wherever temperature it is, to 0 Celsius. This is called the Specific Heat and is 2.04 kiloJoules/kilogram C

Then you need enough energy added to the ice near 0 C to actually melt the ice by increasing the kinetic energy of the water molecules so that their hydrogen bonds weaken, and the water stops acting like a solid. This is called the Latent Heat of Fusion and is 333 kiloJoules/kilogram.

Let's see how this works!

Example 1: You have a 3 kilogram block of ice at a temperature of -20 C. The energy needed to raise it by 20 C to a new temperature of 0 C is Eh = 2.04 kiloJoules/kg C) x 3 kilograms x (20 C) = 2.04 x 3 x 20 = 122 kiloJoules.

Example 2: You have a 3 kilogram block of ice at 0 C and you want to melt it completely into liquid water. This requires Em = 333 kiloJoules/kg x 3 kilograms = 999 kiloJoules.

Example 3: The total energy needed to melt a 3 kilogram block of ice from -20 C to 0C is E = Eh + Em = 122 kiloJoules + 999 kiloJoules = 1,121 kiloJoules.

Problem 1 - On the surface of the satellite Europa (see NASA's Galileo photo above), the temperature of ice is -220 C. What total energy in kiloJoules is required to melt a 100 kilogram block of water ice on its surface? (Note: Calculate Eh and Em separately then combine them to get the total energy.)

Problem 2 - To a depth of 1 meter, the total mass of ice on the surface of Europa is 2.8 x 10^{16} kilograms. How many Joules would be required to melt the entire surface of Europa to this depth? (Note: Calculate Eh and Em separately then combine them to get the total energy. Then convert kiloJoules to Joules)

Problem 3 - The sun produces 4.0 x 10^{26} Joules every second of heat energy. How long would it take to melt Europa to a depth of 1 meter if all of the Sun's energy could be used? (Note: The numbers are BIG, but don't panic!)

Problem 1 - On the surface of the satellite Europa, the temperature of ice is -220 C. What total energy is required to melt a 100 kilogram block of water ice on its surface?

Answer: You have to raise the temperature by 220 C, then
E = 2.04 x 220 x 100 + 333 x 100
= 44,880 + 33,300
= 78,180 kiloJoules.

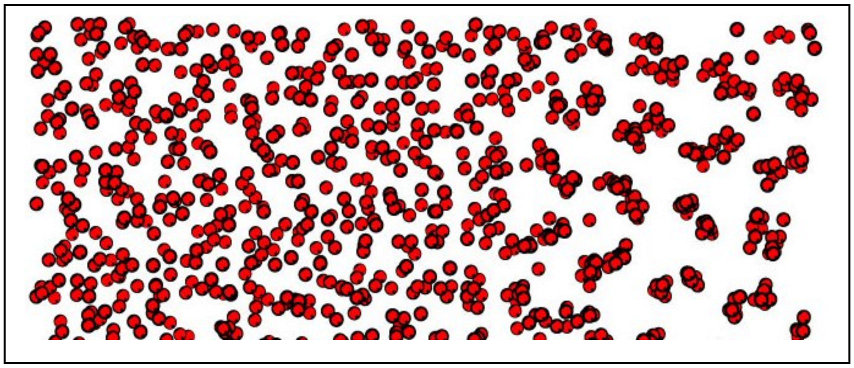
Problem 2 - To a depth of 1 meter, the total mass of ice on the surface of Europa is 2.8 x 10^{16} kilograms. How many joules would be required to melt the entire surface of Europa to this depth?

Note: The radius of Europa is 1,565 km. The surface area is 4 x π x (1,565,000 m)^2 = 3.1 x 10^{13} meters^2. A 1 meter thick shell at this radius has a volume of 3.1 x 10^{13} meters^2 x 1 meter = 3.1 x 10^{13} meters^3. The density of water ice is 917 kilograms/m^3 ,so this ice layer on Europa has a mass of 3.1 x 10^{13} x 917 = 2.8 x 10^{16} kilograms.

Energy = (2.04 x 220 + 333) x 2.8 x 10^{16} kg
= 2.2 x 10^{19} kiloJoules
= 2.2 x 10^{22} Joules.

Problem 3 - The sun produces 4.0 x 10^{26} Joules every second of heat energy. How long would it take to melt Europa to a depth of 1 meter if all of the sun's energy could be used?

Answer: Time = Amount / Rate
= 2.2 x 10^{22} Joules / 4.0 x 10^{26} Joules
= 0.000055 seconds.



This figure is from a super-computer model of the boundary between ice and water. Water molecules in the ice-phase are packed closer together (left-side) than the molecules in liquid water on the right-side. (Courtesy Dr. Jeffry Madura, Duquesne Univ. and Pittsburg Super computer Center.)

To melt a solid, you have to add enough heat energy so that the molecules stop being 'frozen' into a crystalline structure, and begin to move throughout the volume of the material. This is actually a two-step process. First, you have to raise the temperature of the ice until it is near 0 C. Next, you have to add enough extra energy to the ice to make it melt. It takes 2,000 Joules to heat 1 kilogram of ice by 1 C, and it takes another 330,000 Joules to melt 1 kilogram of ice at 0 C to a liquid state. Suppose we had a 2 kilogram block of ice at -50 C and wanted to melt it.

Step 1: First heat the ice to 0 C. This requires Eh = 2,000 Joules x 50 degrees x 2 kilograms = 200,000 Joules.

Step 2: Then melt the ice at 0 C. This takes Em = 330,000 joules x 2 kilograms = 660,000 Joules.

The total energy needed to melt the 2 kilogram block of ice is ET = Eh + Em = 200,000 Joules + 660,000 Joules so ET = **860,000 Joules! This also equals 860 kiloJoules.**

Problem 1: The table below gives the temperature of ice on several different bodies across the solar system. Using the above example as a guide, calculate Em, Eh and Et for each situation. The first entry has been done as a guide.

Problem 2: On which body would it be the most difficult for an astronaut to create liquid water from 1 kilogram of water ice?

Object	Temp. (C)	Ice Mass (Kg)	Eh (kiloJoules)	Em (kiloJoules)	Et (kiloJoules)
Moon (night)	-153	30.0	9,180	9,900	19,080
Mars	-70	200.0			
Europa	-170	5.0			
Enceladus	-200	53.0			
Miranda	-187	25.0			

Problem 1: The table below gives the temperature of ice on several different bodies across the solar system. Using the above example as a guide, calculate Em, Eh and Et for each situation.

Object	Temp. (C)	Ice Mass (Kg)	Eh (kiloJoules)	Em (kiloJoules)	Et (kiloJoules)
Moon (night)	-153	30.0	9,180	9,900	19,080
Mars	-70	200.0	28,000	66,000	94,000
Europa	-170	5.0	1,700	1,650	3,350
Enceladus	-200	53.0	21,200	17,490	38,690
Miranda	-187	25.0	9,350	8,250	17,600

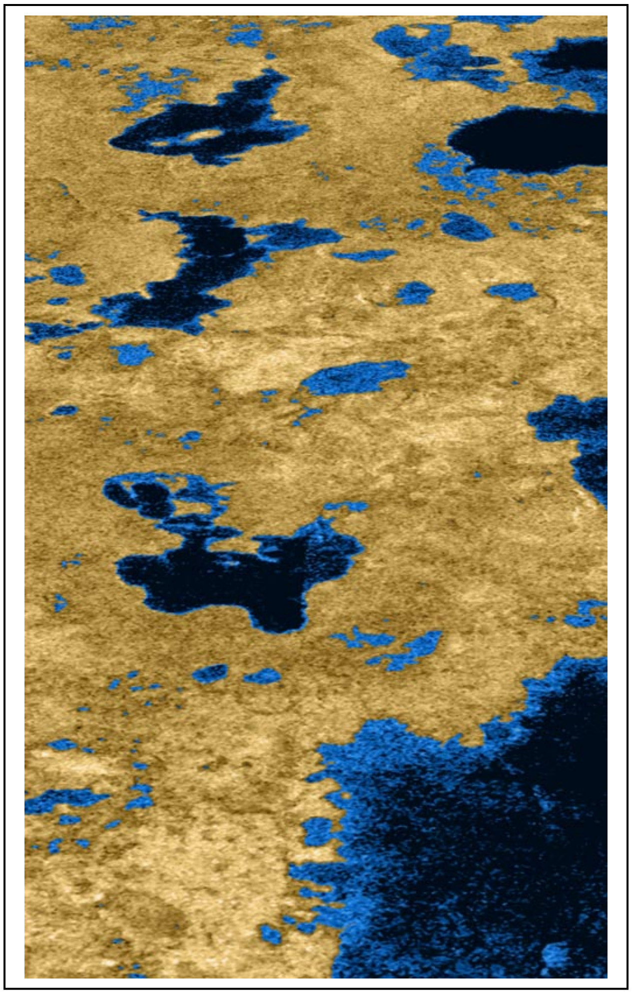
With all units included: For the Moon, students will compute Eh = (2,000 Joules/kg C) x 30 kg x 153 C = 9,180,000 Joules = 9,180 kiloJoules. Em = 330,000 Joules/kg x 30 kg = 9,900,000 Joules = 9,900 kiloJoules. The total is 19,080 kilojoules.

Problem 2: On which body would it be the most difficult for an astronaut to create liquid water from 1 kilogram of water ice?

Answer: Students may look at the table and select Mars because it has the largest total energy requirement, but this number is based on 200 kilograms of ice, not 1 kilogram. Students can re-do the table for an ice mass of 1 kilogram, or they can notice that they will get the same result if they divide each of the ET values by the ice mass in column 3. The result will be that for 1 kilogram Moon = 636 kiloJoules; Mars = 470 kiloJoules; Europa = 670 kiloJoules; Enceladus = 730 kiloJoules and Miranda = 704 KiloJoules and so the answer is Enceladus!

Astronauts will have to provide 730,000 Joules of heat energy to melt each 1 kilogram block of water ice.

Lakes of Methane on Titan



A key goal in the search for life elsewhere in the universe is to detect liquid water, which is generally agreed to be the most essential ingredient for living systems that we know about.

The image to the left is a false-color synthetic radar map of a northern region of Titan taken during a flyby of the cloudy moon by the robotic Cassini spacecraft in July, 2006. On this map, which spans about 150 kilometers across, dark regions reflect relatively little of the broadcast radar signal. Images like this show Titan to be only the second body in the solar system to possess liquids on the surface. In this case, the liquid is not water but methane!

Future observations from Cassini during Titan flybys will further test the methane lake hypothesis, as comparative wind affects on the regions are studied.

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Answer: 150 km / 77 millimeters = **1.9 km/mm**.

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Answer: Combining the areas over the rectangular field of view gives about 1/4 of the area covered. The field of view measures 77 mm x 130mm or 150 km x 247 km or an area of 37,000 km^2. The dark areas therefore cover about 1/4 x 37,000 km^2 or **9,300 km^2.**

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Answer: Volume = area x height = 9,300 km^2 x (0.02 km) = **190 km^3**.

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?

.

Answer: 190 km^3 / 150 km^3 = **1.3 Lake Tahoes**.

As a comet orbits the sun, it produces a long tail stretching millions of kilometers through space. The tail is produced by heated gases leaving the nucleus of the comet.

This image of the head of Comet Tempel-1 was taken by the Hubble Space Telescope on June 30, 2005. It shows the 'coma' formed by these escaping gases about 5 days before its closest approach to the sun (perihelion). The most interesting of these ingredients is ordinary water.

Problem 1 – The NASA spacecraft Deep Impact flew-by Tempel-1 and measured the rate of loss of water from its nucleus. The simple quadratic function below gives the number of tons of water produced every minute, W, as Comet Tempel-1 orbited the sun, where T is the number of days since its closest approach to the sun, called perihelion.

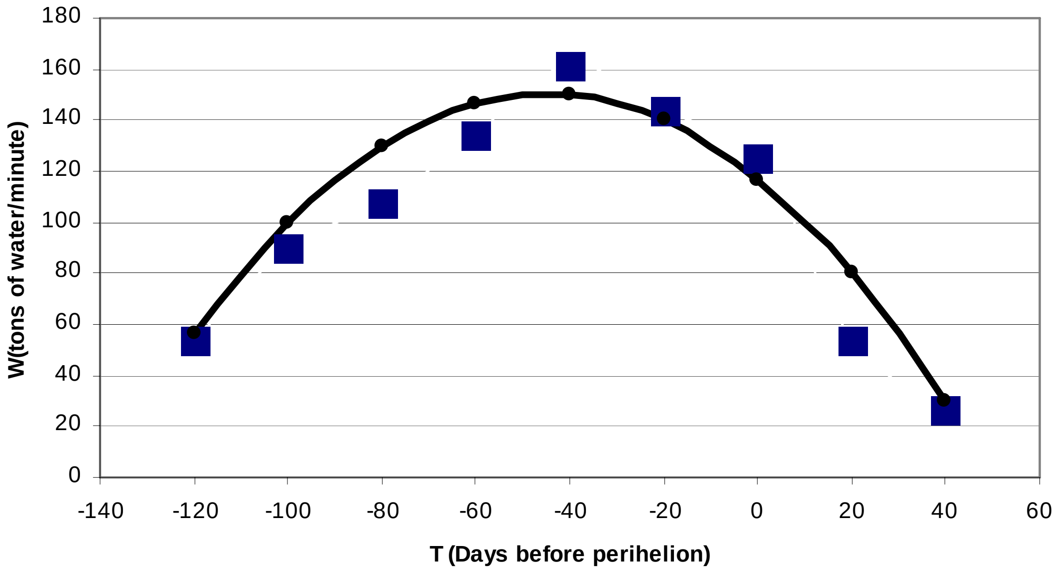
$$W(T) = \frac{(T+140)(60-T)}{60}$$

A) Graph the function W(T). B) For what days, T, will the water loss be zero? C) For what T did the comet eject its maximum amount of water each minute?

Problem 2 – To two significant figures, how many tons of water each minute were ejected by the comet 130 days before perihelion (T = -130)?

Problem 3 - To two significant figures, determine how many tons of water each minute were ejected by the comet 70 days after perihelion (T = +70). Can you explain why this may be a reasonable prediction consistent with the mathematical fit, yet an implausible 'Real World' answer?

Problem 1 – A) The graph below was created with Excel. The squares represent the actual measured data and are shown as an indicator of the quality of the quadratic model fit to the actual data.



Answer: B) The roots of the quadratic equation, where W(T)=0 are for **T=-140** days and **T=+60** days after perihelion. C) The maximum (vertex of the parabola) occurs half-way between the two intercepts at T = (-140+60)/2 or **T = -40** which indicates 40 days before perihelion.

Problem 2 – To two significant figures, how many tons of water each minute were ejected by the comet 130 days before perihelion (T = -130)?

Answer: W(-130) = (-130+140)(60+130)/60 = **32 tons/minute**

Problem 3 - To two significant figures, determine how many tons of water each minute were ejected by the comet 70 days after perihelion (T = +70). Can you explain why this may be a reasonable prediction consistent with the mathematical fit, yet an implausible 'Real World' answer?

Answer: The fitting function W(T) predicts that W(+70) = (70+140)(60-70)/60 = -35 tons per minute. Although this value smoothly follows the prediction curve, it implies that instead of ejecting water (positive answer means a positive rate of change) the comet is absorbing water (negative answer means a negative rate of change), so the prediction is not realistic.

Surrounding every star is a zone within which water can remain in liquid form between a temperature of 0 C (273 K) and +100 C (373 K). The location of this zone is determined by the luminosity of the star. The more luminous the star, the more watts it can deliver on the surface of a distant planet, and the warmer the planet's surface can become. To be in the Habitable Zone, the temperature must be above the freezing point of water, but below its boiling point.

L	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	3.2
0.1	361	255	209	181	162	147	136	128	120	114	109	104	100	97	93	90
0.5	540	382	312	270	242	220	204	191	180	171	163	156	150	144	139	135
1	642	454	371	321	287	262	243	227	214	203	194	185	178	172	166	161
1.5	711	503	410	355	318	290	269	251	237	225	214	205	197	190	184	178
2	764	540	441	382	342	312	289	270	255	242	230	220	212	204	197	191
2.5	808	571	466	404	361	330	305	286	269	255	243	233	224	216	209	202
3	845	598	488	423	378	345	319	299	282	267	255	244	234	226	218	211
3.5	878	621	507	439	393	359	332	311	293	278	265	254	244	235	227	220
4	908	642	524	454	406	371	343	321	303	287	274	262	252	243	235	227
4.5	935	661	540	468	418	382	354	331	312	296	282	270	259	250	242	234
5	960	679	554	480	429	392	363	340	320	304	290	277	266	257	248	240
5.5	983	695	568	492	440	402	372	348	328	311	297	284	273	263	254	246
6	1005	711	580	503	450	410	380	355	335	318	303	290	279	269	260	251
6.5	1025	725	592	513	459	419	388	363	342	324	309	296	284	274	265	256
7	1045	739	603	522	467	426	395	369	348	330	315	302	290	279	270	261

In the table above, the surface temperature of a planet has been calculated. The left-hand column gives the luminosity of the star in multiples of the sun's power, and the columns give the surface temperatures in Kelvin degrees, at distances from 0.2 to 3.2 Astronomical Units. The model assumes a planet with an albedo of A = 0.35 and carbon dioxide concentration of C = 315 ppm similar to Earth.

Problem 1 - For each luminosity, shade-in the distance ranges where the temperatures are close to the liquid water temperature range of 273 to 373 K.

Problem 2 - What do you notice about how the HZs change with luminosity?

Problem 3 - Assume that all planets orbit in the same plane. What is the area of the HZs around the stars with these luminosities? (Give the area in square Astronomical Units times π to 2 significant figures)

Problem 4 - In our solar system (the sun has a luminosity of 1.0 in the table) there are two planets in the HZ. Based on the HZ ring areas, how many planets would you predict would be in the HZ of a star with 7-times the sun's luminosity?

Answer Key

L	Area	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	3.2
0.1	0.12	361	255	209	181	162	147	136	128	120	114	109	104	100	97	93	90
0.5	0.48	540	382	312	270	242	220	204	191	180	171	163	156	150	144	139	135
1	1.08	642	454	371	321	287	262	243	227	214	203	194	185	178	172	166	161
1.5	1.32	711	503	410	355	318	290	269	251	237	225	214	205	197	190	184	178
2	1.92	764	540	441	382	342	312	289	270	255	242	230	220	212	204	197	191
2.5	2.24	808	571	466	404	361	330	305	286	269	255	243	233	224	216	209	202
3	3.00	845	598	488	423	378	345	319	299	282	267	255	244	234	226	218	211
3.5	3.40	878	621	507	439	393	359	332	311	293	278	265	254	244	235	227	220
4	3.40	908	642	524	454	406	371	343	321	303	287	274	262	252	243	235	227
4.5	2.88	935	661	540	468	418	382	354	331	312	296	282	270	259	250	242	234
5	3.80	960	679	554	480	429	392	363	340	320	304	290	277	266	257	248	240
5.5	4.80	983	695	568	492	440	402	372	348	328	311	297	284	273	263	254	246
6	5.28	1005	711	580	503	450	410	380	355	335	318	303	290	279	269	260	251
6.5	5.28	1025	725	592	513	459	419	388	363	342	324	309	296	284	274	265	256
7	6.44	1045	739	603	522	467	426	395	369	348	330	315	302	290	279	270	261

Problem 1 - Answer: **See the shaded areas in the above table. Note, students may show smaller bands if they stick to T < 373 and T > 273 limits, but you may note that to the 0.1 AU binning in the distances, it is OK to round up or down by one distance bin.**

Problem 2 - Answer: **As the star's luminosity increases, the HZ moves farther away from the star, and its width increases.**

Problem 3 - Answer: The area of a circular ring is $A = \pi \times (Ro^2 - Ri^2)$ where Ri is the inner radius and Ro is the outer radius. Example for L = 3 star, Ri = 1.0 AU, Ro = 2.0 AU so $A = \pi \times (Ro^2 - Ri^2) = 1.32\pi$. **Column 2 gives the answers.**

Problem 4 - In our solar system (the sun has a luminosity of 1.0 in the table) there are two planets in the HZ. Based on the HZ ring areas, how many planets would you predict would be in the HZ of a star with 7-times the sun's luminosity?

Answer: This problem can be solved using ratios of the HZ areas in the table.

$$\frac{2 \text{ Planets}}{1.08\pi \text{ AU}^2} = \frac{X \text{ Planets}}{6.44\pi \text{ AU}^2}$$

So X = 2 x (6.44/1.08) = 11.9

So for this star, there could be **11 or 12 planets!** The more luminous a star, the more area there is for the HZ and the more opportunities there are to find habitable planets with liquid water.

Death Stars

Artist rendering courtesy NASA/G. Bacon (STScI)

Our sun is an active star that produces a variety of storms, such as solar flares and coronal mass ejections. Typically, these explosions of matter and radiation are harmless to Earth and its living systems, thanks to our great distance from the sun, a thick atmosphere, and a strong magnetic field. The most intense solar flares rarely exceed about 10^{21} Watts and last for an hour, which is small compared to the sun's luminosity of 3.8×10^{26} Watts.

A long-term survey with the Hubble Space Telescope of 215,000 red dwarf stars for 7 days each revealed 100 'solar' flares during this time. Red dwarf stars are about 1/20000 times as luminous as our sun. Average flare durations were about 15 minutes, and occasionally exceeded 2.0×10^{21} Watts.

Problem 1 - By what percentage does a solar flare on our sun increase the brightness of our sun?

Problem 2 - By what percentage does a stellar flare on an average red dwarf increase the brightness of the red dwarf star?

Problem 3 - Suppose that searches for planets orbiting red dwarf stars have studied 1000 stars for a total of 480 hours each. How many flares should we expect to see in this survey?

Problem 4 - Suppose that during the course of the survey in Problem 3, 5 exoplanets were discovered orbiting 5 of the surveyed red dwarf stars. To two significant figures, about how many years would inhabitants on each planet have to wait between solar flares?

Problem 1 - By what percentage does a solar flare on our sun increase the brightness of our sun?

Answer: $P = 100\% \times (1.0 \times 10^{21}$ Watts$)/(3.8 \times 10^{26}$ Watts$)$

P = 0.00026 %

Problem 2 - By what percentage does a stellar flare on an average red dwarf increase the brightness of the red dwarf star?

Answer: The average luminosity of a red dwarf star is stated as 1/20000 times our sun's luminosity, which is 3.8×10^{26} Watts, so the red dwarf star luminosity is about 1.9×10^{22} Watts.

$P = 100\% \times (2.0 \times 10^{21}$ Watts$)/(1.9 \times 10^{22}$ Watts$)$

P = 10 %

Problem 3 - Suppose that searches for planets orbiting red dwarf stars have studied 1000 stars for a total of 480 hours each. How many flares should we expect to see in this survey?

Answer: We have two samples: N1 = 215,000 stars for 7 days each producing 100 flares. N2 = 1000 stars for 480 hours each, producing x flares.

From the first survey, we calculate a rate of flaring per star per day:

Rate = 100 flares x (1/215,000 stars) x (1 / 7days)

= 0.000066 flares/star/day

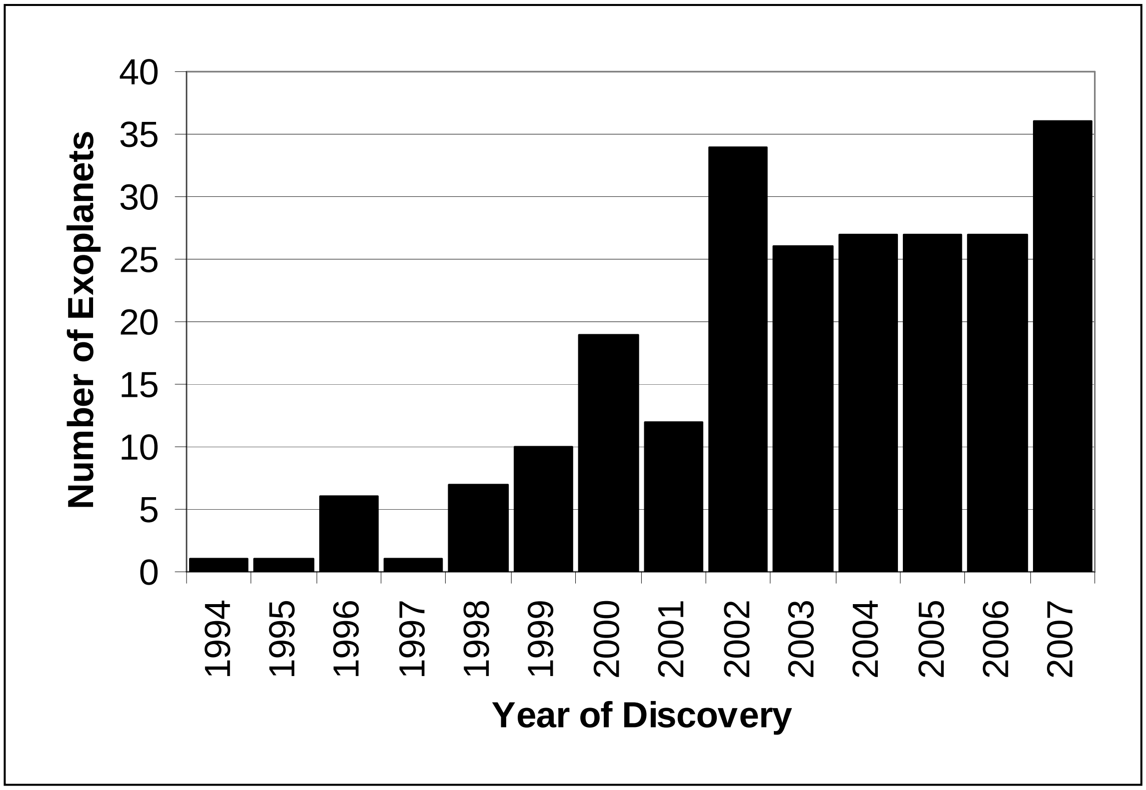
Now we multiply this rate by the size of our current sample to get the number of flares to be seen in the 480-hour (20-day) period.

N = 0.000066 flares/star/day) x (1000 stars) x (20 days)

= 1.32 or **1 flare event**.

Problem 4 - Suppose that during the course of the survey in Problem 3, 5 exoplanets were discovered orbiting 5 of the surveyed red dwarf stars. To two significant figures, about how long would inhabitants on each planet have to wait between solar flares?

Answer: The Flare rate was 0.000066 flares/star/day. For 1 star, we have T = 0.000066 flares/star/day x (1 star) = 0.000066 flares/day so the time between flares (days/flare) is just T = 1/0.000066 = 15,151 days. Since there are 365 days/year, this is about **42 years between flares on the average**.



The 'area under a curve' is an important mathematical quantity that defines virtually all mathematical functions. It has many practical uses as well. For example, the function plotted above, call it P(X), determines the number of new planets, P, that were discovered each year, X, between 1994 through 2007. It was created by tallying-up the number of actual planet discoveries reported in research articles during each of the years. The actual curve representing the function P(X) is shown as a black line, and the columns indicate the number of discoveries per year.

Problem 1 - How would you calculate the total number of planets detected between 1994-2007?

Problem 2 - What is the total area under the curve shown in the figure?

Problem 3 - Suppose N(1995,2000) represents the number of planets detected during the years 1995, 1996, 1997, 1998, 1999, 2000. A) What does N(1994,2007) mean? B) What does N(1994,2007) - N(1994,2000) mean?

Problem 4 - Evaluate:

A) N(2002,2007)
B) N(1999,2002)

Problem 5 - Evaluate and re-write in terms of N (Example for a function defined for X = A,B,C and D : N(A,B) + N(C,D) is just N(A,D))

A) N(1994,2001) + N(2002,2007)
B) N(2001,2005) - N(2002,2005)
C) N(1994,2007) - N(1994,2001)

Answer Key

Problem 1 - How would you calculate the total number of planets detected between 1994-2007? Answer; You would add up the numbers of planets detected during each year, between 1994 and 2007. This is the same as adding up the areas of each of the individual columns.

Problem 2 - What is the total area under the curve shown in the figure? Answer; the total area is found by adding up the numbers for each column: 1 + 1 + 6+ 1 + 7 + 10 + 19 + 13 + 34 + 26 + 27 + 27 + 27 + 36 = **235 planets.**

Problem 3 - Suppose N(1995,2000) represents the number of planets detected during the years 1995, 1996, 1997, 1998, 1999, 2000.

A) What does N(1994,2007) mean? Answer: It means the total number of planets detected between 1994 and 2007, which is **235 planets**

B) What does N(1994,2007) - N(1994,2000) mean? Answer: It means to subtract the number of planets detected between 1995-200 from the total number of planets detected between 1994-2007. N(1994,2000) =1 + 1 + 6+ 1 + 7 + 10 + 19 = 45, so you will get 235 - 45 = **190 planets.**

Problem 5 - Evaluate:

A) N(2002,2007) = 34 + 26 + 27 + 27 + 27 + 36 = **177 planets.**
B) N(1999,2002) = 10 + 19 + 13 + 34 = **76 planets.**

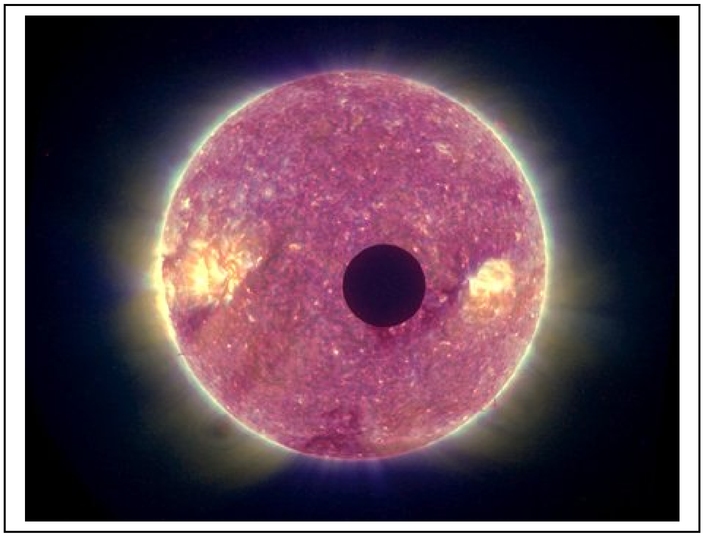
Problem 6 - Evaluate and re-write in terms of N:

A) N(1994,2001) + N(2002,2007) = (1 + 1 + 6+ 1 + 7 + 10 + 19 + 13) + (34 + 26 + 27 + 27 + 27 + 36) = 58 + 177 = **235 planets which is just N(1994,2007)**

B) N(2001,2005) - N(2002,2005) = (13 + 34 + 26 + 27 + 27) - (34 + 26 + 27 + 27) = **13 planets, which is just N(2001, 2001).**

C) N(1994,2007) - N(1994,2001) = (1 + 1 + 6+ 1 + 7 + 10 + 19 + 13 + 34 + 26 + 27 + 27 + 27 + 36) - (1 + 1 + 6+ 1 + 7 + 10 + 19 + 13) = 235 - 58 = **177 planets which is just N(2002,2007)**

Kepler – The Hunt for Earth-like Planets



On March 11, 2009, NASA launched the Kepler satellite. Its 3-year mission is to search 100,000 stars in the constellation Cygnus and detect earth-sized planets. How can the satellite do this?

The image to the left shows what happens when a planet passes across the face of a distant star as viewed from Earth. In this case, this was the planet Mercury on February 25, 2007.

The picture was taken by the STEREO satellite. Notice that Mercury's black disk has reduced the area of the sun. This means that, on Earth, the light from the sun dimmed slightly during the Transit of Mercury. Because Mercury was closer to Earth than the Sun, Mercury's disk appears very large. If we replace Mercury with the Moon, the lunar disk would exactly cover the disk of the Sun and we would have a total solar eclipse.

Now imagine that the Sun was so far away that you couldn't see its disk at all. The light from the Sun would STILL be dimmed slightly. The Kepler satellite will carefully measure the brightness of more than 100,000 stars to detect the slight changes caused by 'transiting exoplanets'.

Problem 1 – With a compass, draw a circle 160-millimeters in radius to represent the sun. If the radius of the sun is 696,000 kilometers, what is the scale of your sun disk in kilometers/millimeter?

Problem 2 – At the scale of your drawing, what would be the radius of Earth (R = 6,378 km) and Jupiter (R = 71,500 km)?

Problem 3 – What is the area of the Sun disk in square millimeters?

Problem 4 – What is the area of Earth and Jupiter in square millimeters?

Problem 5 – By what percent would the area of the Sun be reduced if: A) Earth's disk were placed in front of the Sun disk? B) Jupiter's disk were placed in front of the Sun disk?

Problem 6 – For the transit of a large planet like Jupiter, draw a graph of the percentage brightness of the star (vertical axis) as it changes with time (horizontal axis) during the transit event. Assume that the entire transit takes about 1 day from start to finish.

Answer Key

Problem 1 – With a compass, draw a circle 160-millimeters in radius to represent the sun. If the radius of the Sun is 696,000 kilometers, what is the scale of your Sun disk in kilometers/millimeter? **Answer: 4,350 km/mm**

Problem 2 – At the scale of your drawing, what would be the radius of Earth (R = 6,378 km) and Jupiter (R = 71,500 km)? **Answer: 1.5 mm and 16.4 mm respectively.**

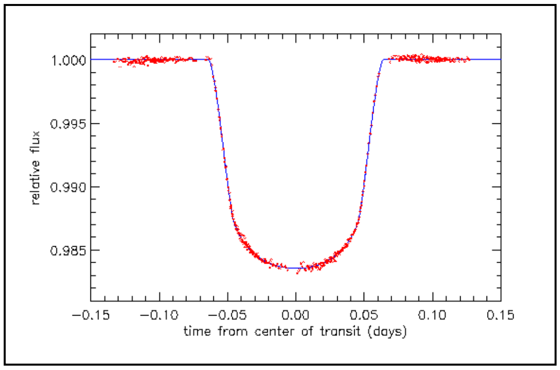
Problem 3 – What is the area of the sun disk in square millimeters? Answer; π x $(160)^2$ **= 80,400 mm^2.**

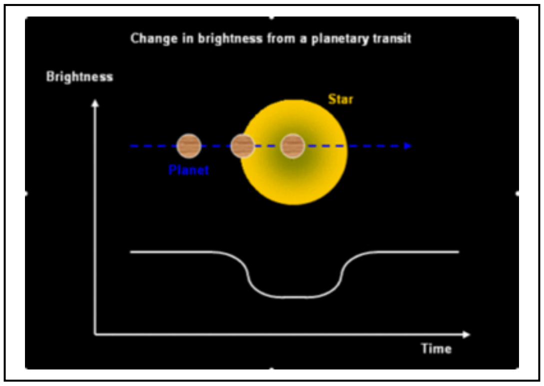
Problem 4 – What is the area of Earth and Jupiter in square millimeters? Answer: Earth = π x $(1.5)^2$ = **7.1 mm^2**. Jupiter = π x $(16.4)^2$ = **844.5 mm^2**.

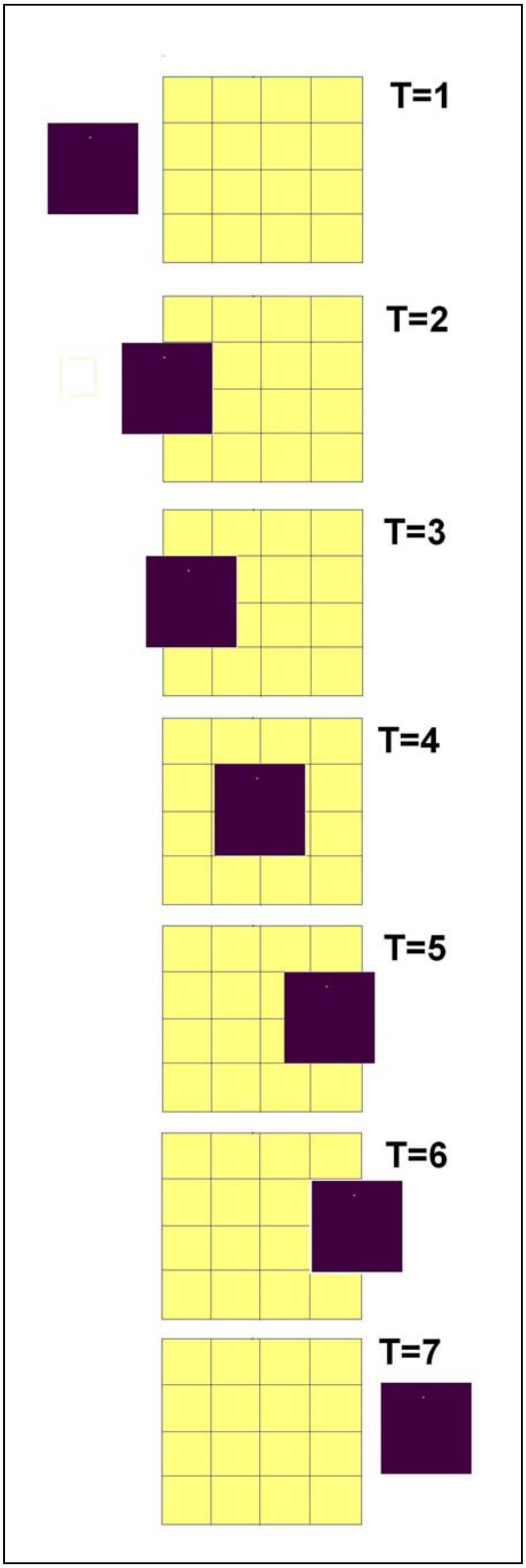
Problem 5 – By what percent would the area of the sun be reduced if: A) Earth's disk were placed in front of the Sun disk? B) Jupiter's disk were placed in front of the Sun disk? Answer A) 100% x (7.1 mm^2 / 80400 mm^2) = 100% x 0.000088 **= 0.0088 %**
B) Jupiter: 100% x (844.5 mm^2 / 80400 mm^2) = 100% x 0.011 = **1.1%.**

Problem 6 – For the transit of a large planet like Jupiter, draw a graph of the percentage brightness of the star (vertical axis) as it changes with time (horizontal axis) during the transit event. Assume that the entire transit takes about 1 day from start to finish.

Answer: Students should note from their answer to Problem 5 that when the planet disk is fully on the star disk, the star's brightness will dim from 100% to 100% - 1.1% = 98.9%. Students should also note that as the transit starts, the stars brightness will dim as more of the planet's disk begins to cover the star's disk. Similarly, as the planet's disk reaches the edge of the star's disk, the area covered by the planet decreases and so the star will gradually brighten to its former 100% level. The figures below give an idea of the kinds of graphs that should be produced. The left figure is from the Hubble Space Telescope study of the star HD209458 and its transiting Jupiter-sized planet.





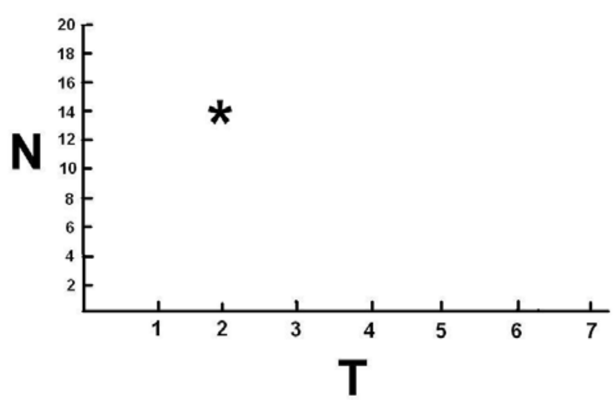


NASA's Kepler spacecraft recently announced the discovery of five new planets orbiting distant stars. The satellite measures the dimming of the light from these stars as planets pass across the face of the star as viewed from Earth. To see how this works, lets look at a simple model.

In the Bizarro Universe, stars and planets are cubical, hot spherical. Bizarro astronomers search for distant planets around other stars by watching planets pass across the face of the stars and cause the light to dim.

Problem 1 - The sequence of figures shows the transit of one such planet, Osiris (black). Complete the 'light curve' for this star by counting the number of exposed 'star squares' not shaded by the planet. At each time, T, create a graph of the number of star brightness squares. The panel for T=2 has been completed and plotted on the graph below.

Problem 2 - If you knew that the width of the star was 1 million kilometers, how could you use the data in the figure to estimate the width of the planet?



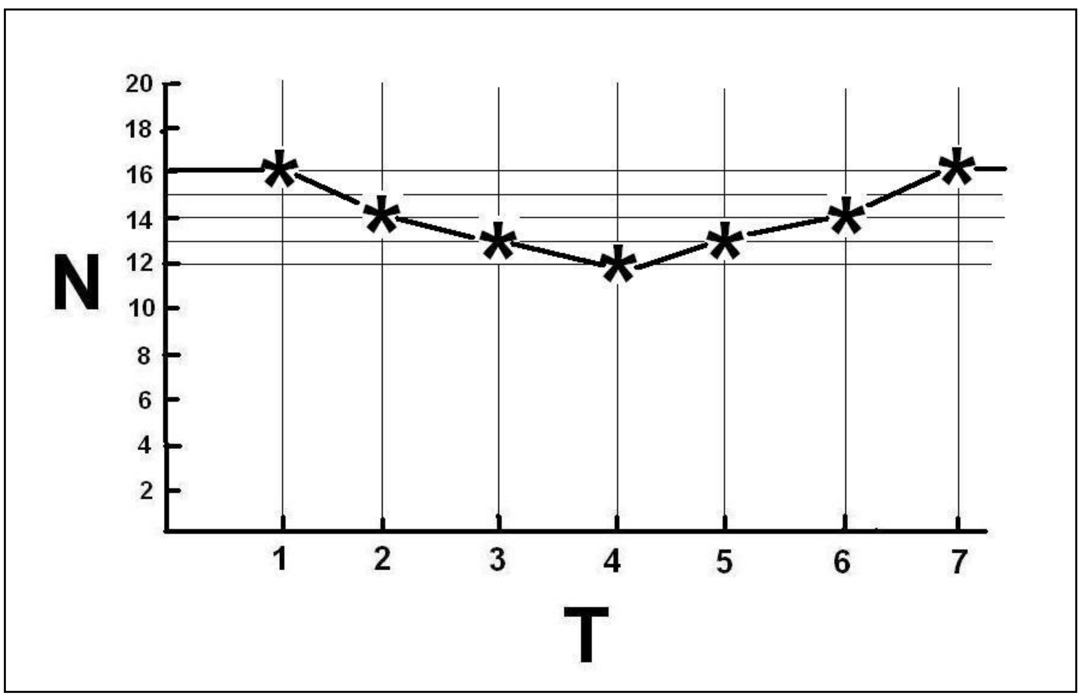
Problem 1 - The sequence of figures shows the transit of one such planet, Osiris (black). Complete the 'light curve' for this star by counting the number of exposed 'star squares' not shaded by the planet. At each time, T, create a graph of the number of star brightness squares. The panel for T=2 has been completed and plotted on the graph below.

Answer: **Count the number of yellow squares in the star and plot these for each value of T in the graph as shown below. Note, for T= 3 and 5, the black square of the planet occupies 2 full squares and 2 half squares for a total of 2 + 1/2 + 1/2 = 3 squares covered, so there are 16 - 3 = 13 squares remaining that are yellow.**

Problem 2 - If you knew that the width of the star was 1 million kilometers, how could you use the data in the figure to estimate the width of the planet?

Answer: The light curve shows that the planet caused the light from the star to decrease from 16 units to 12 units because the planet blocked 16-12 = 4 units of the stars surface area. That means that the planet squares occupy 4/16 of the stars area as seen by the astronomers. The area of the star is just the area of a square, so the area of the square planet is 4/16 of the stars area or Ap = 4/16 x Astar. Since the star as a width of Wstar = 1 million kilometers, the planet will have a width of $Wp = Wstar\sqrt{\frac{4}{16}}$ or **500,000 kilometers**.

The amount of star light dimming is proportional to the ratio of the area of the planet and the star facing the observer. The Kepler satellite can detect changes by as little as 0.0001 in the light from a star, so the smallest planets it can detect have diameters about 1/100 the size of the stars that they orbit. For a star with a diameter of the sun, 1.4 million kilometers, the smallest planet detectable by the Transit Method has a diameter about equal to 14,000 kilometers or about the size of Earth.



Period (days)	F	G	K
0-10	11	138	20
11-20	7	53	16
21-30	4	25	6
31-40	2	13	0
41-50	1	7	0
51-60	1	1	0
61-70	0	1	0
71-80	0	1	0
> 81	0	3	2
Total:	26	242	44

On June 16, 2010 the Kepler mission scientists released their first list of stars that showed evidence for planets passing across the face of their stars. Out of the 156,097 target stars that were available for study, 52,496 were studied during the first 33 days of the mission. Their brightness was recorded every 30 minutes during this time, resulting in over 83 million high-precision measurements.

700 stars had patterns of fading and brightening expected for planet transits. Of these, data were released to the public for 306 of the stars out of a sample of about 88,000 target stars.

The surveyed stars for this study were distributed by spectral class according to F = 8000, G = 55,000 and K = 25000. For the 306 stars, 43 were K-type, 240 were G-type, and 23 were K-type. Among the 312 transits detected from this sample of 306 stars, the table above gives the number of transits detected for each stellar type along with the period of the transit.

Problem 1 - Comparing the F, G and K stars, how did the frequency of the stars with transits compare with the expected frequency of these stars in the general population?

Problem 2 - The distance of the planet from its star can be estimated in terms of the orbital distance of Earth from our sun as $D^3 = T^2$ where D = 1.0 is the distance of Earth from the sun, and T is in multiples of 1 Earth Year. A) What is the distance of Mercury from our sun if its orbit period is 88 days? B) What is the range of orbit distances for the transiting planets in multiples of the orbit of Mercury if the orbit times range from 5 days to 80 days?

Problem 3 - As a planet passes across the star's disk, the star's brightness dims by a factor of 0.001 in brightness. If the radius of the star is 500,000 km, and both the planet and star are approximated as circles, what is the radius of the planet A) in kilometers? B) In multiples of Earth's diameter (13,000 km)?

Problem 1 - Comparing the F, G and K stars, how did the frequency of the stars with transits compare with the expected frequency of these stars in the general population?
Answer: Of the 88,000 stars F = 8000/88000 = 9%; G = 55000/88000 = 63% and K=25000/88000 = 28%.

For the 306 stars: F = 43/306 = 14%; G = 240/306 = 78% and K = 23/306 = 8%....so there were significantly fewer transits detected for K-type stars (8%) compared to the general population (28%).

Note: Sampling error accounts for s = $(306)^{1/2}$ = +/-17 stars or a +/- 6% uncertainty which is not enough to account for this difference in the K-type stars.

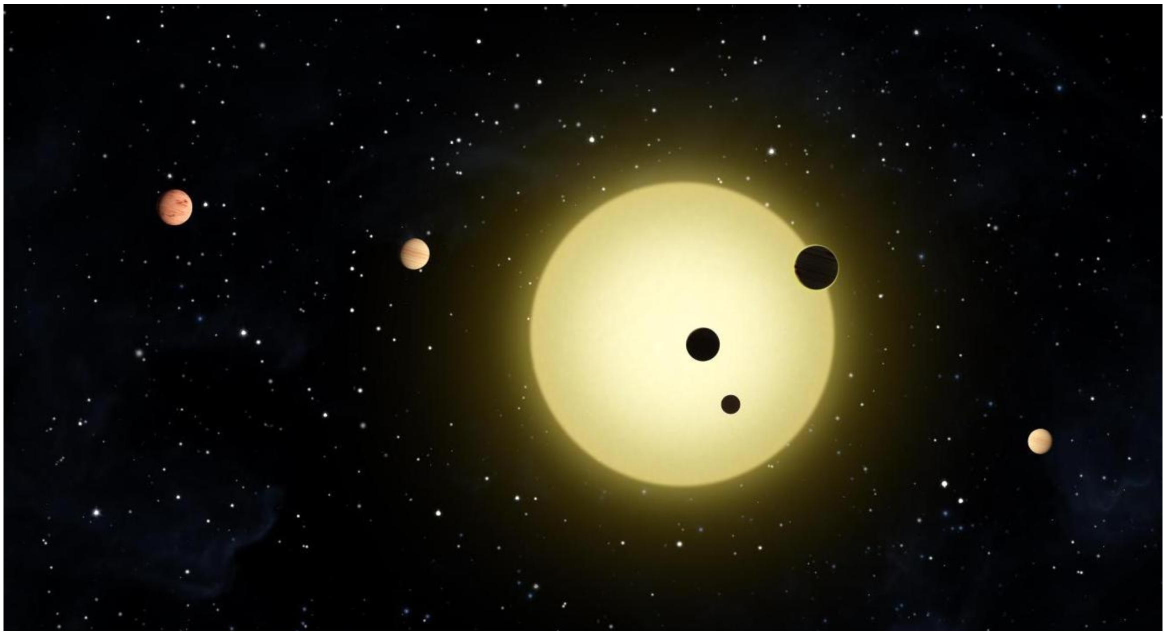
Problem 2 - The distance of the planet from its star can be estimated in terms of the orbital distance of Earth from our sun as $D^3 = T^2$ where D = 1.0 is the distance of Earth from the sun, and T is in multiples of 1 Earth Year. A) What is the distance of Mercury from our sun if its orbit period is 88 days? B) What is the range of orbit distances for the transiting planets in multiples of the orbit of Mercury if the orbit times range from 5 days to 80 days?

Answer: A) T = 88 days/365 days = 0.24 Earth Years, than $D^3 = 0.24^2$, $D^3 = 0.058$, so $D = (0.058)^{1/3}$ and so D = **0.39 times Earth's orbit distance**.
B) The time range is 0.014 to 0.22 Earth Years, and so D is in the range from 0.058 to 0.36 Earth distances. Since Mercury has D = 0.39, in terms of the orbit distance of Mercury, the transiting planets span a range from 0.058/0.39 = **0.15** to 0.36/0.39 = **0.92 Mercury orbits**.

Problem 3 - As a planet passes across the star's disk, the star's brightness dims by a factor of 0.001 in brightness. If the radius of the star is 500,000 km, and both the planet and star are approximated as circles, what is the radius of the planet A) in kilometers? B) In multiples of Earth's diameter (13,000 km)?

Answer: The amount of dimming is equal to the ratio of the areas of the planet's disk to the star's disk, so $0.001 = \pi R^2/\pi(500{,}000)^2$ so **R = 15,800 kilometers**, which equals a diameter of 31,600 kilometers. Since Earth's diameter = 13,000 km, the transiting planet is about **2.4 times the diameter of Earth**.

Earth-like Planets by the Score!



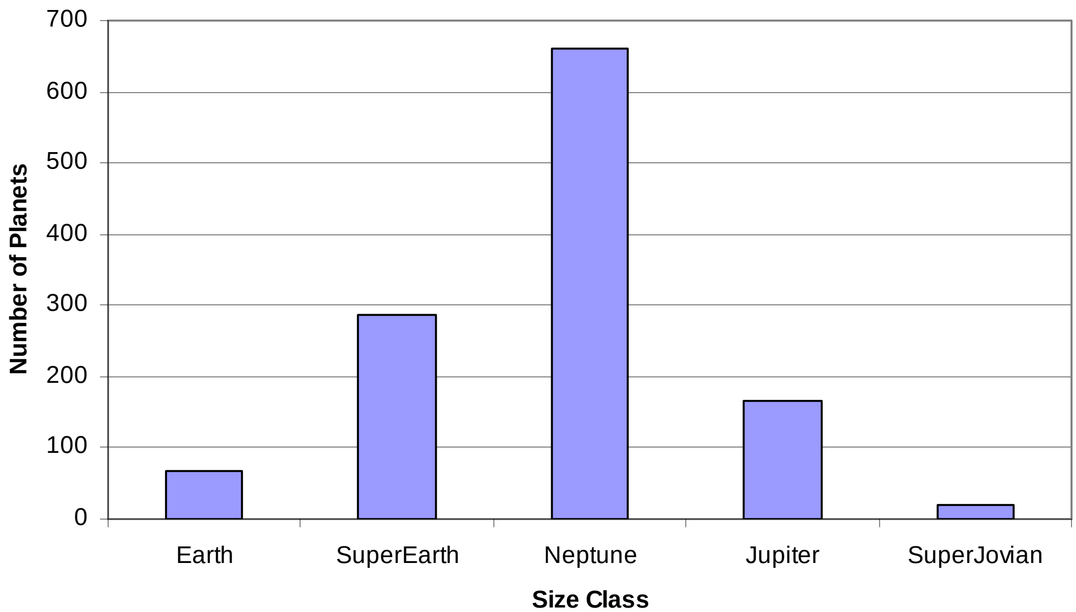
NASA's Kepler Space Observatory recently announced the results of its continuing survey of 156,453 stars in the search for planet transits. Their survey, in progress for just under one year, has now turned up 1,235 transits from among this sample of stars of which 33 were eliminated because they were too big to be true planets. About 30 percent of the remaining candidates belong to multiple-planet systems in which several planets orbit the same star. Among the other important findings are the numbers of planet candidates among the various planet types summarized as follows: Earth-sized = 68; superEarths = 288; Neptune-sized = 662; Jupiter-sized=165; superJovians=19.

Problem 1 - Create a histogram that shows the number of candidate planets among the 5 different size classes.

Problem 2 - What percentage of all the planets detected by Kepler were found to be Earth-sized?

Problem 3 - Extrapolating from the Kepler findings, which was based on a search of 156,453 stars, about how many Earth-sized planets would you expect to find if the Milky Way contains about 40 billion stars similar to the ones surveyed by NASA's Kepler Space Observatory?

Problem 1 - Create a histogram that shows the number of candidate planets among the 5 different size classes. Answer:



Problem 2 - What percentage of all the planets detected by Kepler were found to be Earth-sized? Answer: P = 100% x (68/1202) = **5.7 %**

Problem 3 - Extrapolating from the Kepler findings, which was based on a search of 156,453 stars, about how many Earth-sized planets would you expect to find if the Milky Way contains about 40 billion stars similar to the ones surveyed by Kepler?

Answer: There are 40 billion candidate stars in the Milky Way, so by using simple proportions and re-scaling the survey to the larger sample size

$$\frac{68}{157,453} = \frac{X}{40 billion}$$

we get about (40 billion/156,453) x 68 planets or about **17 million Earth-sized planets**.

Note: *The Kepler survey has not been conducted long enough to detect planets much beyond the orbit of Venus in our own solar system, so in time many more earth-sized candidates farther away from their stars will be reported in the years to come. This means that there may well be considerably more than 17 million Earth-sized planets orbiting stars in the Milky Way similar to our own sun.*

Table of Candidate Planet Sizes

Size Class	Size (Earth Radius)	Number of candidates
Earths	R <1.25	68
Super-Earths	1.25 < R < 2.0	288
Neptunes	2.0 < R < 6.0	662
Jovians	6.0 < R < 15	165
Super-Jovians	15 < R < 22	19
Dwarf Stars, etc	R > 22	15

NASA's Kepler mission has just completed its first year of surveying 156,453 stars to detect the tell-tail signs of distant planets passing across the faces of their stars .This causes a slight dimming of the starlight, which can be detected by the satellite observatory. From the 1,235 transits detected so far, 33 were eliminated because the planets would have been far larger than Jupiter, and possibly dwarf stars. The table above shows the distribution of the remaining planet candidates among the interesting sizes ranges for planets in our own solar system.

About 30 percent of the candidates have been found to belong to multiple-planet systems, with several planets orbiting the same star. To find Earth-like planets where liquid water could be present, astronomers define a Habitable Zone (HZ) surrounding each star where planet surfaces could be warm enough for liquid water. This is roughly between temperatures of 270 to 300 Kelvin. A careful study of the orbits of the planetary candidates discovered 54 candidate planets in the HZs of their stars. Of these, five planets are roughly Earth-sized, and the other 49 planets range from twice the size of Earth to larger than Jupiter.

Problem 1 - What fraction of the candidate planets from the full Kepler survey were found within the HZs of their respective stars?

Problem 2 - What percentage of Earth-sized planets in the full Kepler survey were found in the HZs of the respective stars?

Problem 3 - If there are about 40 billion stars in the Milky Way that are similar to the stars in the Kepler survey, about how many Earth-sized planets would you expect to find in the HZs of there other stars?

Problem 1 - What fraction of the candidate planets from the full Kepler survey were found within the HZs of their respective stars?

Answer: There are 1,202 candidate planets in the larger survey, and 54 found in their HZ so the percentage is P = 100% x (54/1202) = **4.5%**.

Note: *The essay says that 33 candidates were eliminated because they were probably not planets, so 1,235 - 33 = 1,202 planet candidates.*

Problem 2 - What percentage of Earth-sized planets in the full Kepler survey were found in the HZs of the respective stars?

Answer: There were 68 Earth-sized planets found from among 1,202 candidates, and 5 were Earth-sized, so the percentage of Earth-sized planets in their HZs is P = 100% x (5/68) = 7.4%.

So, if you find one Earth-sized planet orbiting a star in the Kepler survey, there is a 7.4% chance that it will be in its HZ so that liquid water can exist.

Problem 3 - If there are about 40 billion stars in the Milky Way that are similar to the stars in the Kepler survey, about how many Earth-sized planets would you expect to find in the HZs of there other stars?

Answer: Students may use simple scaling. The Kepler star sample contains 156,453 stars. It resulted in the discovery, so far, of 5 Earth-sized planets in the Habitable Zones of their respective stars. So

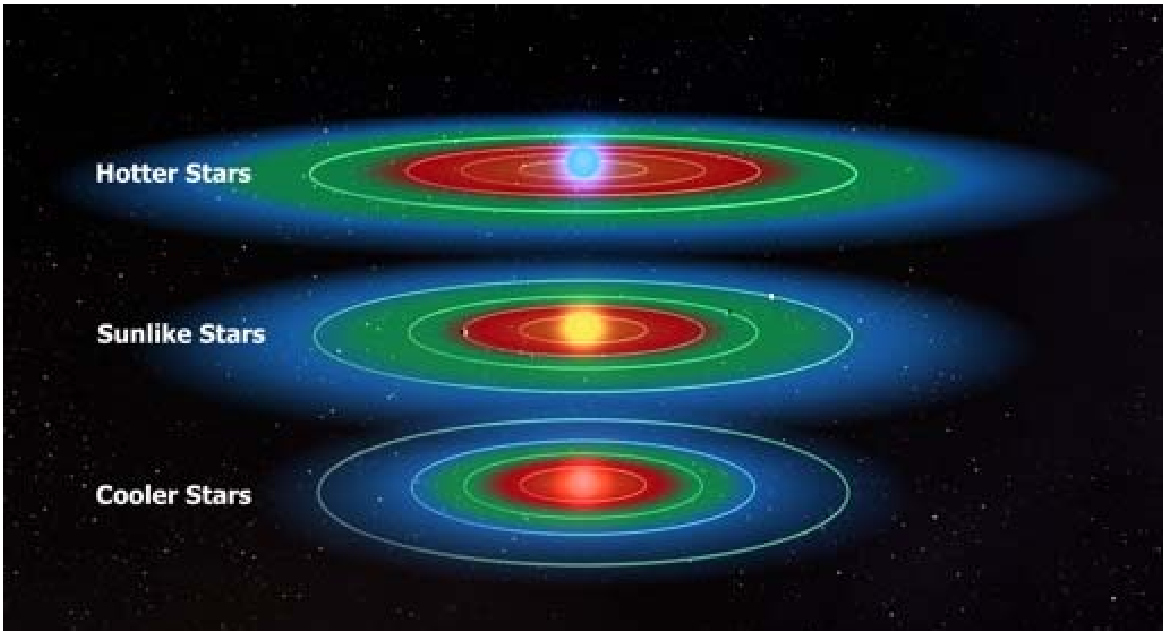
$$\frac{5}{156,453} = \frac{N}{40 billion}$$ so

N = (40 billion / 156,453) x 5 planets

or about **1.3 million planets**.

Another way to think about this is that if you select 30,000 stars in the sky similar to our own sun, you could expect to find about 1 Earth-sized planet orbiting within the Habitable Zone of its star. (5 x 30,000/156453 = 0.96 or about 1.0 planets)

The Goldilocks Planets - Not too hot or cold!



Once you have discovered a planet, you need to figure out whether liquid water might be present. In our solar system, Mercury and Venus are so close to the sun that water cannot remain in liquid form. It vaporizes! For planets beyond Mars, the sun is so far away that water will turn to ice. Only in what astronomers call the Habitable Zone (shown in green in the figure above) will a planet have a chance for being at the right temperature for liquid water to exist in large quantities (oceans) on its surface!

The Table on the following page lists the 54 planets that were discovered by NASA's Kepler Observatory in 2010. These planets come in many sizes as you can see by their radii. The planet radii are given in terms of the Earth, where '1.0' means a planet has a radius of exactly 1 Earth radius (1.0 Re) or 6,378 kilometers. The distance to each planet's star is given in multiples of our Earth-Sun distance, called an Astronomical Unit, so that '1.0 AU' means exactly 150 million kilometers.

Problem 1 - For a planet discovered in its Habitable Zone, and to the nearest whole number, what percentage of planets are less than 4 times the radius of Earth?

Problem 2 - About what is the average temperature of the planets for which R <4.0 Re?

Problem 3 - About what is the average temperature of the planets for which R >4.0 Re?

Problem 4 - Create two histograms of the number of planets in each distance zone between 0.1 and 1.0 AU using bins that are 0.1 AU wide. Histogram-1: for the planets with R > 4.0 Re. Histogram-2 for planets with R < 4.0 Re. Can you tell whether the smaller planets favor different parts of the Habitable Zone than the larger planets?

Problem 5 - If you were searching for Earth-like planets in our Milky Way galaxy, which contains 40 billion stars like the ones studies in the Kepler survey, how many do you think you might find in our Milky Way that are at about the same distance as Earth from its star, about the same size as Earth, and about the same temperature (270 - 290 K) if 157,453 stars were searched for the Kepler survey?

Answer Key

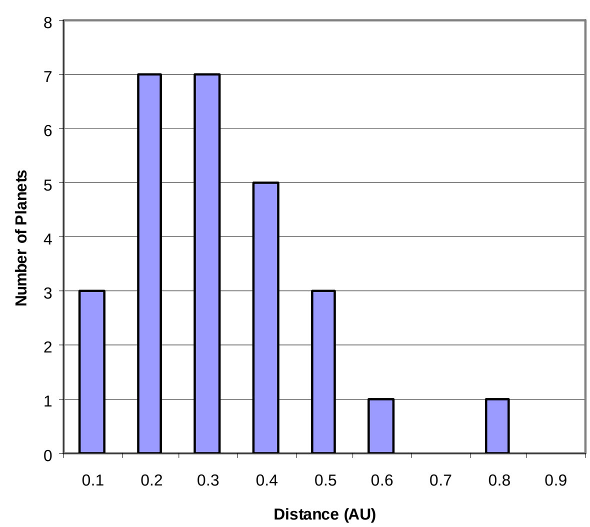
Problem 1 - For a planet discovered in its Habitable Zone, and to the nearest whole number, what percentage of planets are less than 4 times the radius of Earth? Answer: There are 28 planets for which R < 4.0 re, so P = 100% x (28/54) = **52%**

Problem 2 - About what is the average temperature of the planets for which R <4.0 Re? Answer; Students will identify the 28 planets in the table that have R < 4.0, and then average the planet's temperatures in Column 6. **Answer: 317 K**.

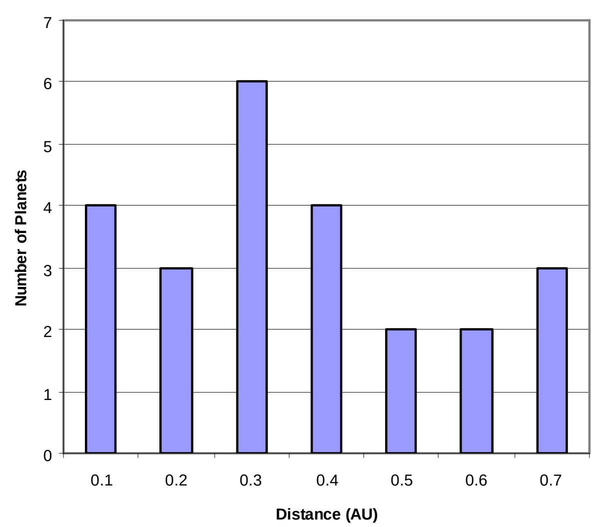
Problem 3 - About what is the average temperature of the planets for which R >4.0 Re? Students will identify the 26 planets in the table that have R > 4.0, and then average the planet's temperatures in Column 6. **Answer: 306 K**.

Problem 4 - Create two histograms of the number of planets in each distance zone between 0.1 and 1.0 AU using bins that are 0.1 AU wide. Histogram-1: for the planets with R > 4.0 Re. Histogram-2 for planets with R < 4.0 Re. Can you tell whether the smaller planets favor different parts of the Habitable Zone than the larger planets? Answer; They tend to be found slightly closer to their stars, which is why in Problem 2 their average temperatures were slightly hotter than the larger planets.

R < 4.0 Re



R > 4.0



Problem 5 - If you were searching for Earth-like planets in our Milky Way galaxy, which contains 40 billion stars like the ones studies in the Kepler survey, how many do you think you might find in our Milky Way that are at about the same distance as Earth from its star, about the same size as Earth, and about the same temperature (270 - 290 K) if 157,453 stars were searched for the Kepler survey?

Answer: Students may come up with a number of different strategies and estimates. For example, they might create Venn Diagrams for the data in the table that meet the criteria given in the problem. Then, from the number of planets in the intersection, find their proportion in the full sample of 54 planets, then multiply this by the ratio of 40 billion to 157,453. Estimates near 1 million are in the right range.

Table of Habitable Zone Candidates

	Planet Name (KOI)	Orbit Period (days)	Distance To Star (AU)	Planet Radius (Re)	Planet Temp. (K)	Star Temp. (K)
1	683.01	278	0.84	4.1	239	5,624
2	1582.01	186	0.63	4.4	240	5,384
3	1026.01	94	0.33	1.8	242	3,802
4	1503.01	150	0.54	2.7	242	5,356
5	1099.01	162	0.57	3.7	244	5,665
6	854.01	56	0.22	1.9	248	3,743
7	433.02	328	0.94	13.4	249	5,237
8	1486.01	255	0.80	8.4	256	5,688
9	701.03	122	0.45	1.7	262	4,869
10	351.01	332	0.97	8.5	266	6,103
11	902.01	84	0.32	5.7	270	4,312
12	211.01	372	1.05	9.6	273	6,072
13	1423.01	124	0.47	4.3	274	5,288
14	1429.01	206	0.69	4.2	276	5,595
15	1361.01	60	0.24	2.2	279	4,050
16	87.01	290	0.88	2.4	282	5,606
17	139.01	225	0.74	5.7	288	5,921
18	268.01	110	0.41	1.8	295	4,808
19	1472.01	85	0.37	3.6	295	5,455
20	536.01	162	0.59	3.0	296	5,614
21	806.01	143	0.53	9.0	296	5,206
22	1375.01	321	0.96	17.9	300	6,169
23	812.03	46	0.21	2.1	301	4,097
24	865.01	119	0.47	5.9	306	5,560
25	351.02	210	0.71	6.0	309	6,103
26	51.01	10	0.06	4.8	314	3,240
27	1596.02	105	0.42	3.4	316	4,656
28	416.02	88	0.38	2.8	317	5,083
29	622.01	155	0.57	9.3	327	5171
30	555.02	86	0.38	2.3	331	5,218
31	1574.01	115	0.47	5.8	331	5,537
32	326.01	9	0.05	0.9	332	3,240
33	70.03	78	0.35	2.0	333	5,342
34	1261.01	133	0.52	6.3	335	5,760
35	1527.01	193	0.67	4.8	337	5,470
36	1328.01	81	0.36	4.8	338	5,425
37	564.02	128	0.51	5.0	340	5,686
38	1478.01	76	0.35	3.7	341	5,441
39	1355.01	52	0.27	2.8	342	5,529
40	372.01	126	0.50	8.4	344	5,638
41	711.03	125	0.49	2.6	345	5,488
42	448.02	44	0.21	3.8	346	4,264
43	415.01	167	0.61	7.7	352	5,823
44	947.01	29	0.15	2.7	353	3,829
45	174.01	56	0.27	2.5	355	4,654
46	401.02	160	0.59	6.6	357	5,264
47	1564.01	53	0.28	3.1	360	5,709
48	157.05	118	0.48	3.2	361	5,675
49	365.01	82	0.37	2.3	363	5,389
50	374.01	173	0.63	3.3	365	5,829
51	952.03	23	0.12	2.4	365	3,911
52	817.01	24	0.13	2.1	370	3,905
53	847.01	81	0.37	5.1	372	5,469
54	1159.01	65	0.30	5.3	372	4,886

Life-size model of the Webb Space Telescope.

In 2014, the new Webb Space Telescope will be launched. This telescope, designed to detect distant sources of infrared 'heat' radiation, will be a powerful new instrument for discovering distant dwarf planets far beyond the orbit of Neptune and Pluto.

Scientists are already predicting just how sensitive this new infrared telescope will be, and the kinds of distant bodies it should be able to detect in each of its many infrared channels. This problem shows how this forecasting is done.

Problem 1 - The angular diameter of an object is given by the formula:

$$\theta(R) = 0.0014\frac{L}{R} \text{ arcseconds}$$

Create a single graph that shows the angular diameter, $\theta(R)$, for an object the size of dwarf planet Pluto (L=2,300 km) spanning a distance range, R, from 30 AU to 100 AU. How big will Pluto appear to the telescope at a distance of 90 AU (about 3 times its distance of Pluto from the sun)?

Problem 2 - The temperature of a body that absorbs 40% of the solar energy falling on it is given by

$$T(R) = \frac{250}{\sqrt{R}}$$

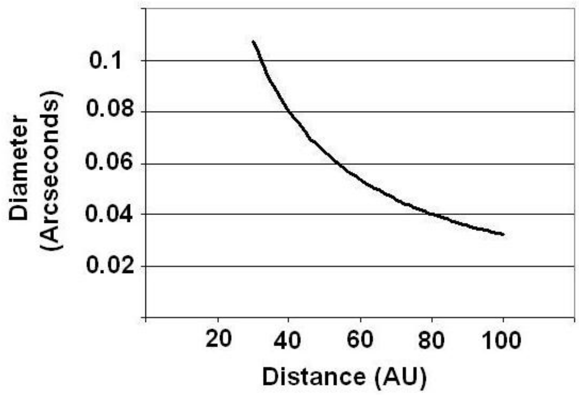
where R is the distance from the sun in AU. Create a graph that shows $\theta(R)$ vs R for objects located in the distance range from 30 to 100 AU. What will be the predicted temperature of a Pluto-like object at 90 AU?

Problem 3 - A body with an angular size $\theta(R)$ given in arcseconds emits 40% of its light energy in the infrared and has a temperature given by T(R) in Kelvin degrees. Its brightness in units of Janskys, F, at a wavelength of 25 microns (2500 nanometers) will be given by:

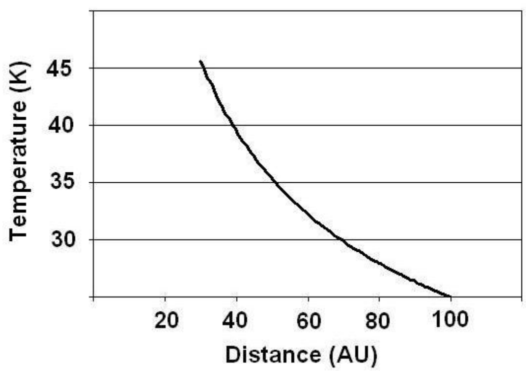
$$F(T) = \frac{11500}{(e^x - 1)}\theta(R)^2 \quad \text{Janskys} \qquad \text{where} \qquad x = \frac{580}{T(R)}$$

From the formula for $\theta(R)$ and T(R), create a curve F(R) for a Pluto-like object. If the Webb Space Telescope cannot detect objects fainter than 4 nanoJanskys, what will be the most distant location for a Pluto-like body that this telescope can detect? (Hint: Plot the curve with a linear scale in R and a log10 scale in F.)

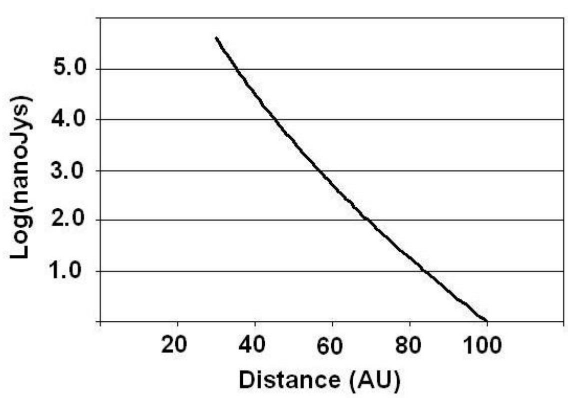
Problem 1 - Answer: At 90 AU, the disk of a Pluto-sized body will be 0.035 arcseconds in diameter.



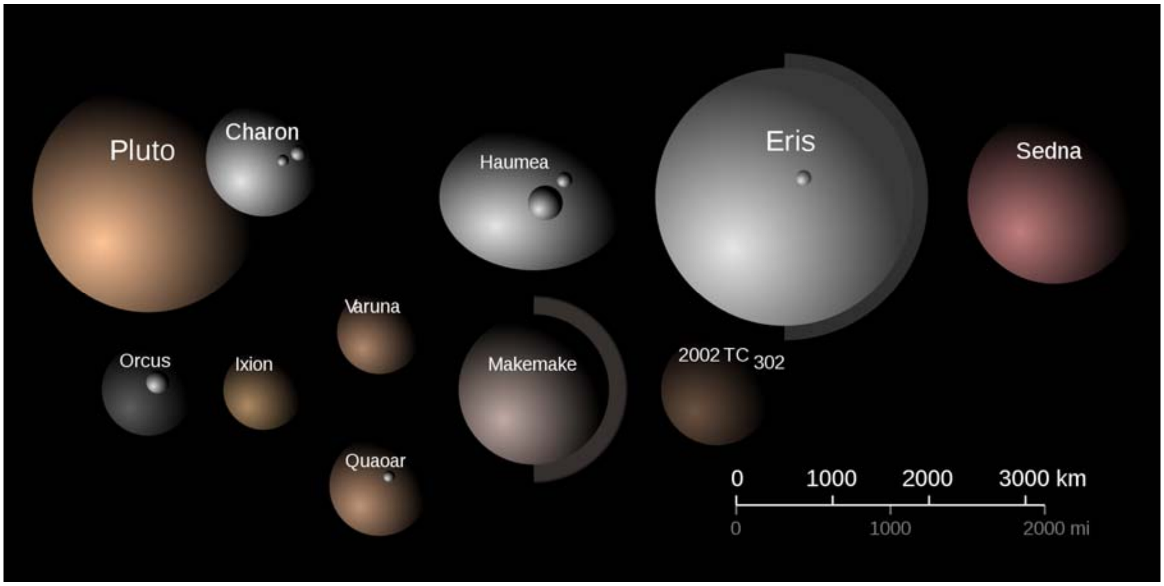
Problem 2 Answer; At 90 AU, the predicted temperature will be about 27 K.



Problem 3 - Answer; At 4 nanoJanskies, Log(4 nanoJy) = 0.60 which occurs at a distance of about 90 AU.



Dwarf Planets and Kepler's Third Law



Object	Distance (AU)	Period (years)
Mercury	0.4	0.24
Venus	0.7	0.61
Earth	1.0	1.0
Mars	1.5	1.88
Ceres	2.8	4.6
Jupiter	5.2	11.9
Saturn	9.5	29.5
Uranus	19.2	84.0
Neptune	30.1	164.8
Pluto	39.4	247.7
Ixion	39.7	
Huya	39.8	
Varuna	42.9	
Haumea	43.3	285
Quaoar	43.6	
Makemake	45.8	310
Eris	67.7	557
1996-TL66	82.9	
Sedna	486.0	

Astronomers have detected over 500 bodies orbiting the sun well beyond the orbit of Neptune. Among these 'Trans-Neptunian Bodies (TNOs) are a growing number that rival Pluto in size. This caused astronomers to rethink how they should define the term 'planet'.

In 2006 Pluto was demoted from a planet to a dwarf planet, joining the large asteroid Ceres in that new group. Several other TNOs also joined that group, which now includes five bodies shown highlighted in the table. A number of other large objects, called Plutoids, are also listed.

Problem 1 - From the tabulated data, graph the distance as a function of period on a calculator or Excel spreadsheet. What is the best-fit: A) Polynomial function? B) Power-law function?

Problem 2 - Which of the two possibilities can be eliminated because it gives unphysical answers?

Problem 3 - Using your best-fit model, what would you predict for the periods of the TNOs in the table?

Problem 1 - From the tabulated data, graph the distance as a function of period on a calculator or Excel spreadsheet. What is the best-fit:

A) Polynomial function? **The N=3 polynomial $D(x) = -0.0005x^3 + 0.1239x^2 + 2.24x - 1.7$**

B) Power-law function? **The N=1.5 powerlaw: $D(x) = 1.0x^{1.5}$**

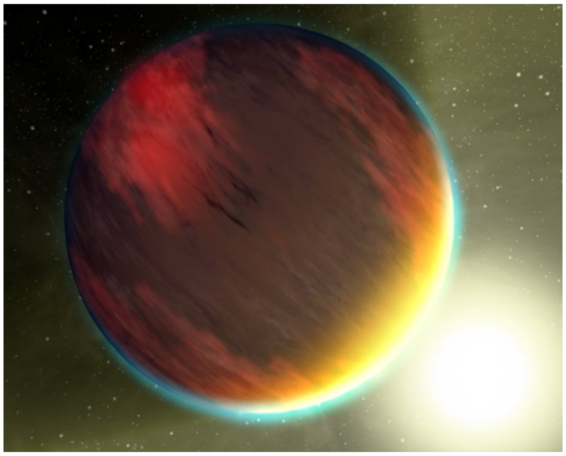
Problem 2 - Which of the two possibilities can be eliminated because it gives unphysical answers? The two predictions are shown in the table:

Object	Distance	Period	N=3	N=1.5
Mercury	0.4	0.24	-0.79	0.25
Venus	0.7	0.61	-0.08	0.59
Earth	1	1	0.66	1.00
Mars	1.5	1.88	1.93	1.84
Ceres	2.8	4.6	5.53	4.69
Jupiter	5.2	11.9	13.22	11.86
Saturn	9.5	29.5	30.33	29.28
Uranus	19.2	84	83.44	84.13
Neptune	30.1	164.8	164.34	165.14
Pluto	39.4	247.7	248.31	247.31
Ixion	39.7		251.21	250.14
Huya	39.8		252.19	251.09
Varuna	42.9		282.94	280.99
Haumea	43.3	285	286.99	284.93
Quaoar	43.6		290.05	287.89
Makemake	45.8	310	312.75	309.95
Eris	67.7	557	562.67	557.04
1996-TL66	82.9		750.62	754.80
Sedna	486		-27044.01	10714.07

Answer: The N=3 polynomial gives negative periods for Mercury, Venus and Sedna, and poor answers for Earth, Mars, Ceres and Jupiter compared to the N=3/2 power-law fit. The N=3/2 power-law fit is the result of Kepler's Third Law for planetary motion which states that the cube of the distance is proportional to the square of the period so that when all periods and distances are scaled to Earth's orbit, Period = Distance$^{3/2}$

Problem 3 - See the table above for shaded entries

Organic Molecules Detected on Distant Planet!



The basic chemistry for life has been detected in a second hot gas planet, HD 209458b, depicted in this artist's concept. Two of NASA's Great Observatories – the Hubble Space Telescope and Spitzer Space Telescope, yielded spectral observations that revealed molecules of carbon dioxide, methane and water vapor in the planet's atmosphere. HD 209458b, bigger than Jupiter, occupies a tight, 3.5-day orbit around a sun-like star about 150 light years away in the constellation Pegasus. (NASA Press release October 20, 2009)

Some Interesting Facts: The distance of the planet from the star HD209458 is 7 million kilometers, and its orbit period (year) is only 3.5 days long. At this distance, the temperature of the outer atmosphere is about 1,000 C (1,800 F). At these temperatures, water, methane and carbon dioxide are all in gaseous form. It is also known to be losing hydrogen gas at a ferocious rate, which makes the planet resemble a comet! The planet itself has a mass that is 69% that of Jupiter, and a volume that is 146% greater than that of Jupiter. The unofficial name for this planet is Osiris.

Problem 1 - The mass of Jupiter is 1.9 x 10^{30} grams. The radius of Jupiter is 71,500 kilometers. A) What is the volume of Jupiter in cubic centimeters, assuming it is a perfect sphere? B) What is the density of Jupiter in grams per cubic centimeter (cc), based on its mass and your calculated volume?

Problem 2 - From the information provided; A) What is the volume of Osiris in cubic centimeters, if it is in the shape of a perfect sphere? B) What is the mass of Osiris in grams? C) What is the density of Osiris in grams/cc, and how does this compare to the density of Jupiter?

Problem 3 - The densities of some common ingredients for planets are as follows:

Rock	3 grams/cc	Ice	1 gram/cc
Iron	9 grams/cc	Mixture of hydrogen + helium	0.7 grams/cc
Water	5 grams/cc		

Based on the average density of Osiris, from what substances do you think the planet is mostly composed?

Problem 1 - The mass of Jupiter is 1.9×10^{30} grams. The radius of Jupiter is 71,500 kilometers.
A) What is the volume of Jupiter in cubic centimeters, assuming it is a perfect sphere?
Answer: The radius of Jupiter, in centimeters, is
R = 71,500 km x (100,000 cm/1 km)
= 7.15×10^{9} cm.

For a sphere, $V = 4/3\, \pi R^3$ so the volume of Jupiter is
$V = 1.33 \times (3.141) \times (7.15 \times 10^{9})^3$
$V = 1.53 \times 10^{30}\ \text{cm}^3$

B) What is the density of Jupiter in grams per cubic centimeter (cc), based on its mass and your calculated volume?
Answer: Density = Mass/Volume so the density of Jupiter is D = (1.9×10^{30} grams)/($1.53 \times 10^{30}\ \text{cm}^3$) = 1.2 gm/cc

Problem 2 - From the information provided;
A) What is the volume of Osiris in cubic centimeters, if it is in the shape of a perfect sphere?
Answer: The information says that the volume is 146% greater than Jupiter so it will be V =
$V=1.46 \times (1.53 \times 10^{30}\ \text{cm}^3)$
= $2.23 \times 10^{30}\ \text{cm}^3$

B) What is the mass of Osiris in grams?
Answer: the information says that it is 69% of Jupiter so
$M = 0.69 \times (1.9 \times 10^{30}$ grams)
= 1.3×10^{30} grams
C) What is the density of Osiris in grams/cc, and how does this compare to the density of Jupiter?
Answer: D = Mass/volume
= 1.3×10^{30} grams / $2.23 \times 10^{30}\ \text{cm}^3$
= 0.58 grams/cc

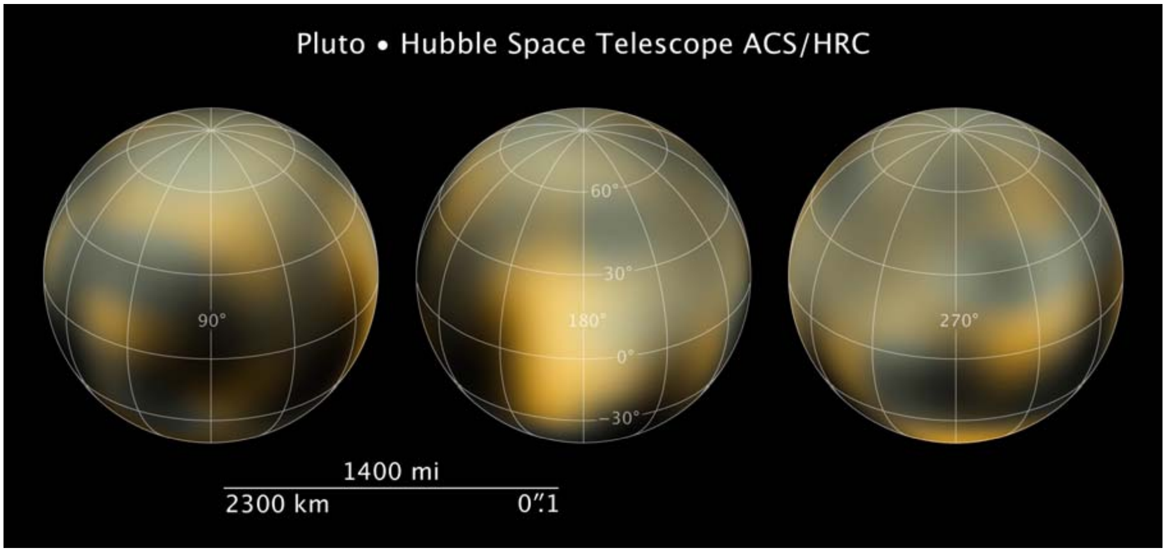
Problem 3 - The densities of some common ingredients for planets are as follows:

Rock	3 grams/cc	Ice	1 gram/cc
Iron	9 grams/cc	Mixture of hydrogen + helium	0.7 grams/cc
Water	5 grams/cc		

Based on the average density of Osiris, from what substances do you think the planet is mostly composed?

Answer: Because the density of Osiris is only about 0.6 grams/cc, the closest match would be a mixture of hydrogen and helium. This means that, rather than a solid planet like earth, which is a mixture of higher-density materials such as iron, rock and water, Osiris has much in common with Jupiter which is classified by astronomers as a Gas Giant!

Seeing a Dwarf Planet Clearly: Pluto



Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. The images, created at the very limits of Hubble's resolving power, show enigmatic light and dark regions that are probably organic compounds (dark areas) and methane or water-ice deposits (light areas). Since these photos are all that we are likely to get until NASA's New Horizons spacecraft arrives in 2015, let's see what we can learn from the image!

Problem 1 - Using a millimeter ruler, what is the scale of the Hubble image in kilometers/millimeter?

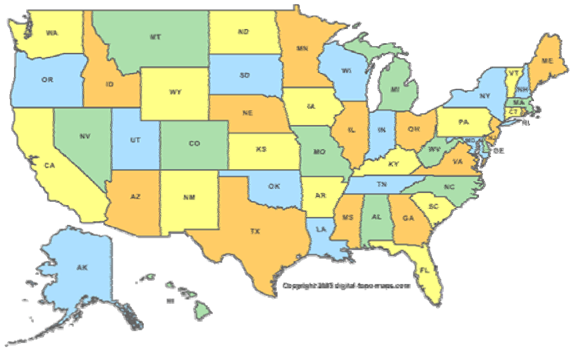
Problem 2 - What is the largest feature you can see on any of the three images, in kilometers, and how large is this compared to a familiar earth feature or landmark such as a state in the United States?

Problem 3 - The satellite of Pluto, called Charon, has been used to determine the total mass of Pluto. The mass determined was about 1.3 x 10^{22} kilograms. From clues in the image, calculate the volume of Pluto and determine the average density of Pluto. How does it compare to solid-rock (3000 kg/m^3), water-ice (917 kg/m^3)?

Inquiry: Can you create a model of Pluto that matches its average density and predicts what percentage of rock and ice may be present?

Problem 1 - Using a millimeter ruler, what is the scale of the Hubble image in kilometers/millimeter? Answer: The Legend bar indicates 2,300 km and is 43 millimeters long so the scale is 2300/43 = **53 km/mm**.

Problem 2 - What is the largest feature you can see on any of the three images, in kilometers, and how large is this compared to a familiar earth feature or landmark such as a state in the United States?
Answer; Student's selection will vary, but on the first image to the lower right a feature measures about 8 mm in diameter which is 8 mm x (53 km/1mm) = **424 kilometers wide**. This is about the same size as the **state of Utah**!

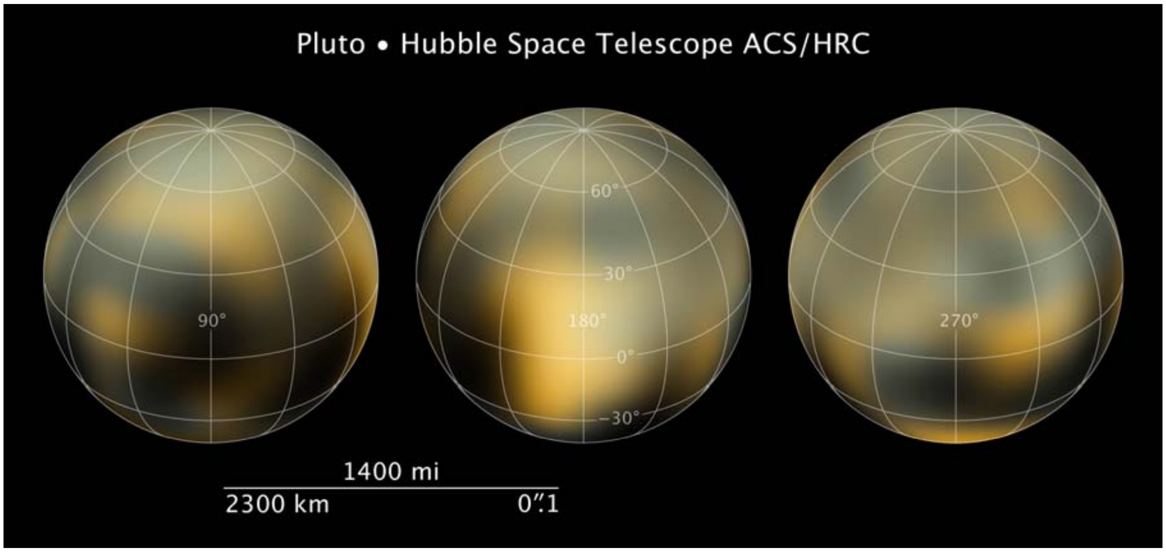


Problem 3 - The satellite of Pluto, called Charon, has been used to determine the total mass of Pluto. The mass determined was about 1.3 x 10^{22} kilograms. From clues in the image, calculate the volume of Pluto and determine the average density of Pluto. How does it compare to solid-rock (3000 kg/m^3), water-ice (917 kg/m^3)? Answer: From the image, Pluto is a sphere with a diameter of 2,300 km, so its volume will be $V = 4/3\ \pi\ (1{,}250{,}000)^3 = 8.2 \times 10^{18}$ meters^3. Then its density is just D = M/V = (1.3 x 10^{22} kilograms)/ (8.2 x 10^{18} meters^3) so **D = 1,600 kg/m^3**. This would be about the density of **a mixture of rock and water-ice**.

Inquiry: Can you create a model of Pluto that matches its average density and predicts what percentage of rock and ice may be present?

Answer: We want to match the density of Pluto (1,600 km/m^3) by using ice (917 kg/m^3) and rock (2300 kg/m^3). Suppose we made Pluto out of half-rock and half-ice **by mass**. The volume this would occupy would be V = (0.5* 1.3 x 10^{22} kilograms/917 kg/m^3) = 7.1x10^{18} meters^3 for the ice, and V = (0.5* 1.3 x 10^{22} kilograms/3000 kg/m^3) = 2.2 x10^{18} meters^3 for the rock, for a total volume of 9.3 x 10^{18} meters^3 for both. This is a bit larger then the actual volume of Pluto (8.2 x 10^{18} meters^3) so we have to increase the mass occupied by ice, and lower the 50% by mass occupied by the rock component. The result, from student trials and errors should yield after a few iterations **about 40% ice and 60% rock.** This can be done very quickly using an Excel spreadsheet. For advanced students, it can also be solved exactly using a bit of algebra.

The Changing Atmosphere of Pluto



Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. Let's see how this is possible!

Problem 1 - The equation for the orbit of Pluto can be approximated by the formula $2433600=1521x^2+1600y^2$. Determine from this equation, expressed in Standard Form, A) the semi-major axis, a; B) the semi-minor axis, b; C) the ellipticity of the orbit, e; D) the longest distance from a focus called the aphelion; E) the shortest distance from a focus, called the perihelion. (Note: All units will be in terms of Astronomical Units. 1 AU = distance from the Earth to the Sun = 1.5×10^{11} meters).

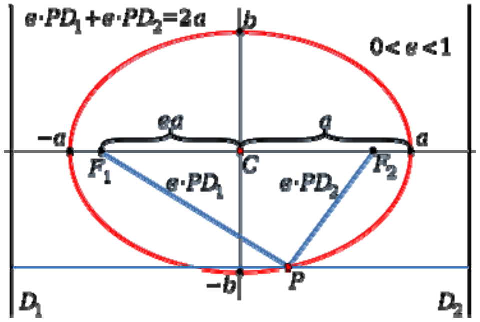
Problem 2 - The temperature of the methane atmosphere of Pluto is given by the formula

$$T(R) = \left(\frac{L(1-A)}{16\pi\sigma R^2} \right)^{\frac{1}{4}} \quad \text{degrees Kelvin (K)}$$

where L is the luminosity of the sun ($L=4 \times 10^{26}$ watts); σ is a constant with a value of 5.67×10^{-8}, R is the distance from the sun to Pluto in meters; and A is the albedo of Pluto. The albedo of Pluto, the ability of its surface to reflect light, is about A = 0.6. From this information, what is the predicted temperature of Pluto at A) perihelion? B) aphelion?

Problem 3 - If the thickness, H, of the atmosphere in kilometers is given by H(T) = 1.2 T with T being the average temperature in degrees K, can you describe what happens to the atmosphere of Pluto between aphelion and perihelion?

Problem 1 - Answer:



In Standard Form $2433600=1521x^2+1600y^2$ becomes $1 = \frac{x^2}{1600} + \frac{y^2}{1521} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

Then A) **a = 40 AU** and B) **b=39 AU**. C) The ellipticity **$e = (a^2 - b^2)^{1/2}/a = 0.22$**. D) The longest distance from a focus is just a(1 + e) = 40(1+0.22) = **49 AU**. E) The shortest distance is just a(1-e) = (1-0.22)(40) = **31 AU**. Written out in meters we have a= 6×10^{12} meters; b= 5.8×10^{12} meters; aphelion = 7.35×10^{12} meters and perihelion = 4.6×10^{12} meters.

Problem 2 - Answer: For R in terms of AU, the formula simplifies to

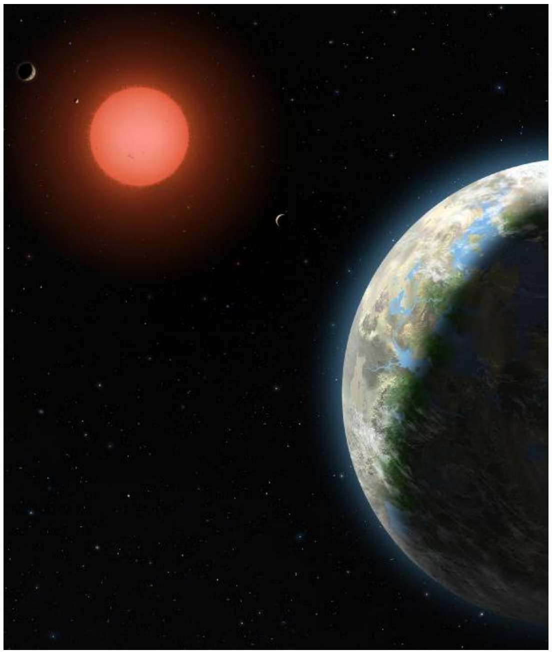
$$T(R) = \left(\frac{4x10^{26}(1-0.6)}{16(3.14)(5.67x10^{-8})(1.5x10^{11})^2 R^2} \right)^{\frac{1}{4}} \quad \text{so } T(R) = \frac{223}{\sqrt{R}} \text{ degrees K}$$

A) For a perihelion distance of 31 AU we have T = $223/(31)^{1/2}$ = 40 K; B) At an aphelion distance of 49 AU we have T = $223/(49)^{1/2}$ = 32 K. Note: The actual temperatures are about higher than this and average about 50K.

Problem 3 - Answer: At aphelion, the height of the atmosphere is about H=1.2x(32) = 38 kilometers, and at perihelion it is about H=1.2x(40) = 48 kilometers, so as Pluto orbits the sun its atmosphere increases and decreases in thickness.

Note: In fact, because the freezing point of methane is 91K, at aphelion most of the atmosphere freezes onto the surface of the dwarf planet, and at aphelion it returns to a mostly gaseous state. This indicates that the simple physical model used to derive H(T) was incomplete and did not account for the freezing-out of an atmospheric constituent.

The Earth-like Planet Gliese 581g



Professors Steven Vogt at UC Santa Cruz, and Paul Butler of the Carnegie Institution have just announced the discovery of a new planet orbiting the nearby red dwarf star Gliese 518. The star is located 20 light years from Earth in the constellation Libra. The planet joins five others in this crowded planetary system, and has a mass about three to four times Earth, making it in all likelihood a rocky planet, rather than a gas giant. The planet is tidally locked to its star which means that during its 37 day orbit, it always shows the same face to the star so that one hemisphere is always in daylight while the other is in permanent nighttime.

One of the most important aspects to new planets is whether they are in a distance zone where water can remain a liquid on the planets surface. The Habitable Zone (HZ) location around a star depends on the amount of light energy that the star produces. For the Sun, the HZ extends from about the orbit of Venus to the orbit of Mars. For stars that emit less energy, the HZ will be much closer to the star. Once an astronomer knows what kind of star a planet orbits, they can calculate over what distances the HZ will exist.

Problem 1 - What is the pattern that astronomers use to name the discovered planets outside our solar system?

Problem 2 - One Astronomical Unit (AU) is the distance between Earth and the Sun (150 million kilometers). Draw a model of the Gliese 581 planetary system with a scale of 0.01 AU per centimeter, and show each planet with a small circle drawn to a scale of 5,000 km/millimeter, based on the data in the table below:

Planet	Discovery Year	Distance (AU)	Period (days)	Diameter (km)
Gliese 581 b	2005	0.04	5.4	50,000
Gliese 581 c	2007	0.07	13.0	20,000
Gliese 581 d	2007	0.22	66.8	25,000
Gliese 581 e	2009	0.03	3.1	15,000
Gliese 581 f	2010	0.76	433	25,000
Gliese 581 g	2010	0.15	36.6	20,000

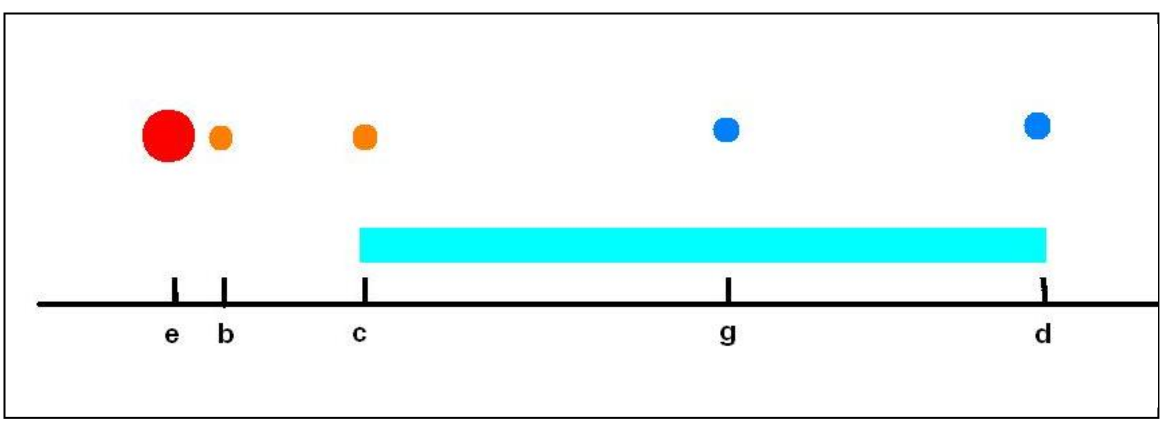
Problem 3 - The Habitable Zone for our solar system extends from 0.8 to 2.0 AU, while for Gliese 581 it extends from about 0.06 to 0.23 because the star shines with nearly 1/100 the amount of light energy as our sun. In the scale model diagram, shade-in the range of distances where the HZ exists for the Gliese 581 planetary system. Why do you think astronomers are excited about Gliese 581g?

Problem 1 - What is the pattern that astronomers use to name the discovered planets outside our solar system? Answer: **According to the order of discovery date.** Note: Gliese 581 A is the designation given to the star itself.

Problem 2 - Draw a model of the Gliese 581 planetary system with a scale of 0.01 AU per centimeter, and show each planet with a small circle drawn to a scale of 5,000 km/millimeter, based on the data in the table. Answer: **The table below gives the dimensions on the scaled diagram. See figure below for an approximate appearance. On the scale of the figure below, Gliese 581f would be located about 32 centimeters to the right of Gliese 581d.**

Planet	Discovery Year	Distance (cm)	Period (days)	Diameter (mm)
Gliese 581 b	2005	4	5.4	10
Gliese 581 c	2007	7	13.0	4
Gliese 581 d	2007	22	66.8	5
Gliese 581 e	2009	3	3.1	3
Gliese 581 f	2010	76	433	5
Gliese 581 g	2010	15	36.6	4

Problem 3 - The Habitable Zone for our solar system extends from 0.8 to 2.0 AU, while for Gliese 581 it extends from about 0.06 to 0.23 because the star shines with nearly 1/100 the amount of light energy as our sun. In the scale model, shade-in the range of distances where the HZ exists. Why do you think astronomers are excited about Gliese 581g? Answer: **See the bar spanning the given distances. Note that Gliese c, g and d are located in the HZ of Gliese 581. Because Gliese 581 g is located near the center of this zone and is very likely to be warm enough for there to be liquid water, which is an essential ingredient for life. Gliese 581c may be too hot and Gliese 581 d may be too cold.**




Artist view of planet G. Bacon (STScI/NASA)

Every 4 days, this planet orbits a sun-like star located 153 light years from Earth. Astronomers using NASA's Hubble Space Telescope have confirmed that this gas giant plane is orbiting so close to its star its heated atmosphere is escaping into space.

Observations taken with Hubble's Cosmic Origins Spectrograph (COS) suggest powerful stellar winds are sweeping the cast-off atmospheric material behind the scorched planet and shaping it into a comet-like tail. COS detected the heavy elements carbon and silicon in the planet's super-hot, 2,000-degree-

Problem 1 - Based upon a study of the spectral lines of hydrogen, carbon and silicon, the estimated rate of atmosphere loss may be as high as 4×10^{11} grams/sec. How fast is it losing mass in: A) metric tons per day? B) metric tons per year?

Problem 2 - The mass of the planet is about 60% of Jupiter, and its radius is about 1.3-times that of Jupiter. If the mass of Jupiter is 1.9×10^{27} kg, and its radius is 7.13×10^{7} meters, what is the density of A) Jupiter? B) HD209458b?

Problem 3 - Suppose that, like Jupiter, the planet has a rocky core with a mass of 18 times Earth. If Earth's mass is 5.9×10^{24} kg, what is the mass of the atmosphere of HD209458b?

Problem 4 - About how long would it take for HD209458b to completely lose its atmosphere at the measured mass-loss rate?

Problem 1 - Based upon a study of the spectral lines of hydrogen, carbon and silicon, the estimated rate of atmosphere loss may be as high as 4×10^{11} grams/sec. How fast is it losing mass in: A) metric tons per day? B) metric tons per year?

Answer: A) 4×10^{11} grams/sec x (10^{-6} kg/gm x 86,400 sec/day = **3.5×10^{10} tons/day**
B) 3.5×10^{10} tons/day x 365 days/year = **1.3×10^{13} tons/year**

Problem 2 - The mass of the planet is about 60% of Jupiter, and its radius is about 1.3-times that of Jupiter. If the mass of Jupiter is 1.9×10^{27} kg, and its radius is 7.13×10^{7} meters, what is the density of A) Jupiter? B) HD209458b?

Answer: A) $V = 4/3 \pi R^3$ so V(Jupiter) = 1.5×10^{24} meters^3. Density = mass/volume so Density (Jupiter) = 1.9×10^{27} kg/1.5×10^{24} meters^3 = **1266 kg/meter^3**.
B) Mass = 0.6 M(Jupiter) and volume = $(1.3)^3$ V(Jupiter) so density = $0.6/(1.3)^3$ x 1266 kg/meter^3 =0.27x1266 = **342 kg/meter^3**.

Problem 3 - Suppose that, like Jupiter, the planet has a rocky core with a mass of 18 times Earth. If Earth's mass is 5.9×10^{24} kg, what is the mass of the atmosphere of HD209458b? Answer: M(HD209458b) = 0.6x Jupiter = 1.1×10^{27} kg so M(atmosphere) = 1.1×10^{27} kg – 18x(5.9×10^{24} kg) = **9.9×10^{26} kg**.

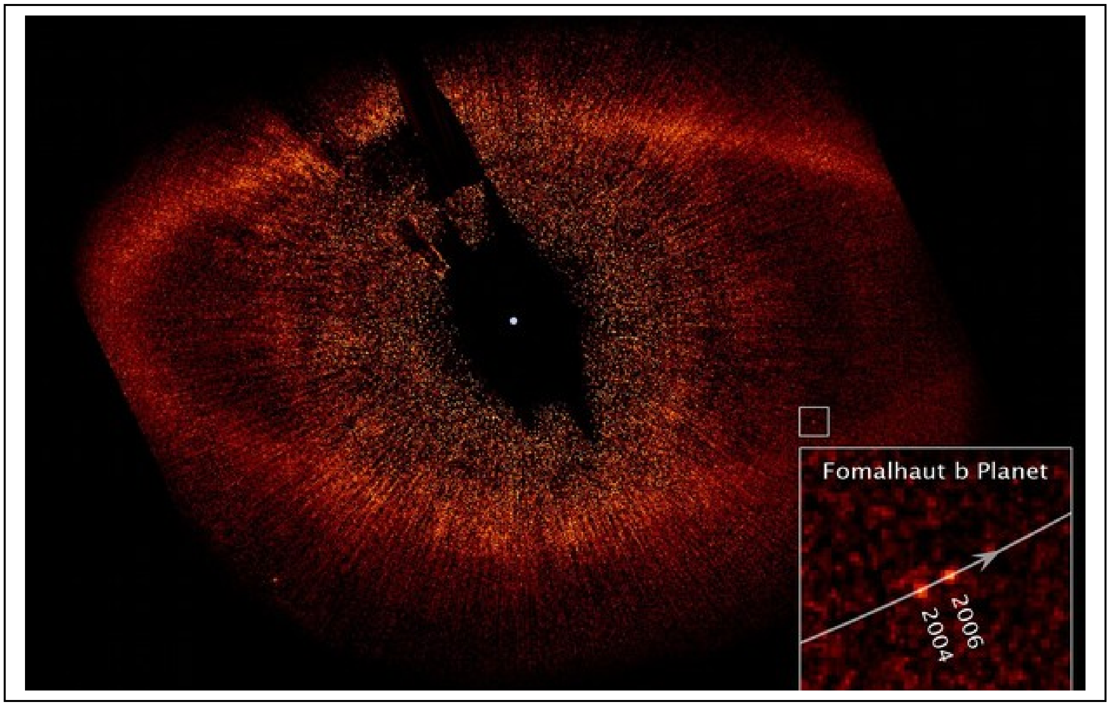
Problem 4 - About how long would it take for HD209458b to completely lose its atmosphere at the measured mass-loss rate?

Answer: Time = Mass/rate
= 9.9×10^{26} kg / (1.3×10^{16} kg/year)
= **7.6×10^{10} seconds**
= 2,456 years!

Hubble Sees a Distant Planet

The bright star Fomalhaut, in the constellation Piscis Austrinus (The Southern Fish) is only 25 light years away. It is 2000^{o} K hotter than the Sun, and nearly 17 times as luminous, but it is also much younger: Only about 200 million years old. Astronomers have known for several decades that it has a ring of dust (asteroidal material) in orbit 133 AU from the star and about 25 AU wide. Because it is so close, it has been a favorite hunting ground in the search for planets beyond our solar system. In 2008 such a planet was at last discovered using the Hubble Space Telescope. It was the first direct photograph of a planet beyond our own solar system.

In the photo below, the dusty ring can be clearly seen, but photographs taken in 2004 and 2006 revealed the movement of one special 'dot' that is now known to be the star's first detected planet. The small square on the image is magnified in the larger inset square in the lower right to show the location of the planet in more detail.



Problem 1 – The scale of the image is 2.7 AU/millimeter. If 1.0 AU = 150 million kilometers, how far was the planet from the star in 2006?

Problem 2 – How many kilometers had the planet moved between 2004 and 2006?

Problem 3 – What was the average speed of the planet between 2004 and 2006 if 1 year = 8760 hours?

Problem 4 – Assuming the orbit is circular, with the radius found from Problem 1, about how many years would it take the planet to make a full orbit around its star?

Answer Key

Problem 1 – The scale of the image is 2.7 AU/millimeter. If 1.0 AU = 150 million kilometers, how far was the planet from the star in 2006?

Answer: The distance from the center of the ring (location of star in picture) to the center of the box containing the planet is 42 millimeters, then 42 x 2.7 AU/mm = 113 AU. Since 1 AU = 150 million km, the distance is 113 x 150 million **= 17 billion kilometers.**

Problem 2 – How many kilometers had the planet moved between 2004 and 2006?

Answer: On the main image, the box has a width of 4 millimeters which equals 4 x 2.7 = 11 AU. The inset box showing the planet has a width of 36 mm which equals 11 AU so the scale of the small box is 11AU/36 mm = 0.3 AU/mm. The planet has shifted in position about 4 mm, so this corresponds to 4 x 0.3 = **1.2 AU or 180 million km**.

Problem 3 – What was the average speed of the planet between 2004 and 2006 if 1 year = 8760 hours?

Answer: The average speed is 180 million km/17520 hours = **10,273 km/hr.**

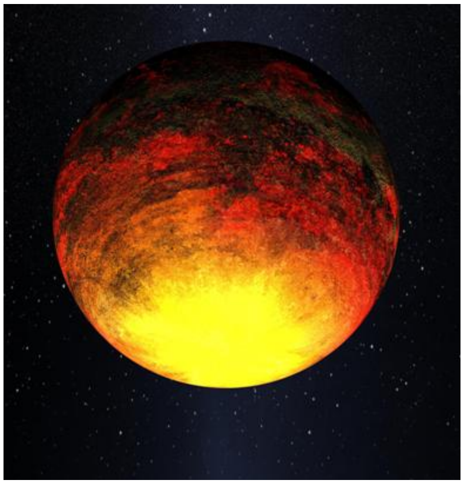
Problem 4 – Assuming the orbit is circular, with the radius found from Problem 1, about how many years would it take the planet to make a full orbit around its star?

Answer: The radius of the circle is 113 AU so the circumference is $2 \pi R$ = 2 (3.141) (113 AU) = 710 AU. The distance traveled by the planet in 2 years is, from Problem 2, about 1.2 AU, so in 2 years it traveled 1.2/710 = 0.0017 of its full orbit. That means a full orbit will take 2.0 years/0.0017 = **1,176 years.**

Note - Because we are only seeing the 'projected' motion of the planet along the sky, the actual speed could be faster than the estimate in Problem 3, which would make the estimate of the orbit period a bit smaller than what students calculate in Problem 4.

A careful study of this system by its discoverer, Dr. Paul Kalas (UC Berkeley) suggests an orbit distance of 119 AU, and an orbit period of 872 years.

Planet Kepler-10b: A matter of gravity



NASA Artist rendition of the sizzling-hot, Earth-like world: Kepler 10b.

The Kepler Space Observatory recently detected an Earth-sized planet orbiting the star Kepler-10. The more than 8 billion year old star, located in the constellation Draco, is 560 light years from Earth. The planet orbits its star at a distance of 2.5 million km with a period of 20 hours, so that its surface temperature exceeds 2,500 F.

Careful studies of the transit of this planet across the face of its star indicates a diameter 1.4 times that of Earth, and an estimated average density of 8.8 grams/cc, which is about that of solid iron, and 3-times the density of Earth's silicate-rich surface rocks.

Problem 1 -Assume that Kepler-10b is a spherical planet, and that the radius of Earth is 6,378 kilometers. What is the total mass of this planet if its density is 8800 kg/meter^3?

Problem 2 - The acceleration of gravity on a planet's surface is given by the Newton's formula

$$a = 6.67x10^{-11} \frac{M}{R^2} \quad \text{meters/sec}^2$$

Where R is distance from the surface of the planet to the planet's center in meters, and M is the mass of the planet in kilograms. What is the acceleration of gravity at the surface of Kepler-10b?

Problem 3 - The acceleration of gravity at Earth's surface is 9.8 meters/sec^2. If this acceleration causes a 68 kg human to have a weight of 150 pounds, how much will the same 68 kg human weigh on the surface of Kepler-10b if the weight in pounds is directly proportional to surface acceleration?

Problem 1 -Assume that Kepler-10b is a spherical planet, and that the radius of Earth is 6,378 kilometers. What is the total mass of this planet if its density is 8800 kg/meter^3?

Answer: The planet is 1.4 times the radius of Earth, so its radius is 1.4 x 6,378 km = 8,929 kilometers. Since we need to use units in terms of meters because we are given the density in cubic meters, the radius of the planet becomes 8,929,000 meters.

Volume = $\frac{4}{3}\pi R^3$

so V = 1.33 x (3.141) x (8,929,000 meters)^3

V = 2.98 x 10^{21} meter^3

Mass = Density x Volume

= 8,800 x 2.98 x 10^{21}

= **2.6x10^{25} kilograms**

Problem 2 - The acceleration of gravity on a planet's surface is given by the Newton's formula

$$a = 6.67x10^{-11}\frac{M}{R^2} \quad \text{meters/sec}^2$$

Where R is distance from the surface of the planet to the planet's center in meters, and M is the mass of the planet in kilograms. What is the acceleration of gravity at the surface of Kepler-10b?

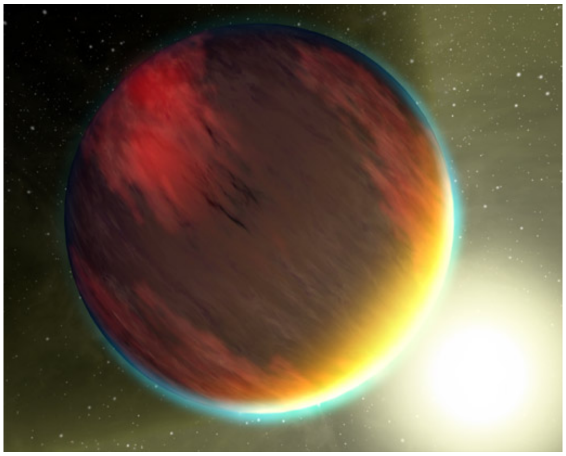
Answer: a = 6.67 x 10^{-11} (2.6 x 10^{25})/(8.929x10^6)^2

= **21.8 meters/sec^2**

Problem 3 - The acceleration of gravity at Earth's surface is 9.8 meters/sec^2. If this acceleration causes a 68 kg human to have a weight of 150 pounds, how much will the same 68 kg human weigh on the surface of Kepler-10b if the weight in pounds is directly proportional to surface acceleration?

Answer: The acceleration is 21.8/9.8 = 2.2 times Earth's gravity, and since weight is proportional to gravitational acceleration we have the proportion:

$\frac{21.8}{9.8} = \frac{X}{150lb}$ and so the human would weigh 150 x 2.2 = **330 pounds!**



The basic chemistry for life has been detected in a second hot gas planet, HD 209458b, depicted in this artist's concept. Two of NASA's Great Observatories – the Hubble Space Telescope and Spitzer Space Telescope, yielded spectral observations that revealed molecules of carbon dioxide, methane and water vapor in the planet's atmosphere. HD 209458b, bigger than Jupiter, occupies a tight, 3.5-day orbit around a sun-like star about 150 light years away in the constellation Pegasus. (NASA Press release October 20, 2009)

Some Interesting Facts: The distance of the planet from the star HD209458 is 7 million kilometers, and its orbit period (year) is only 3.5 days long. At this distance, the temperature of the outer atmosphere is about 1,000 C (1,800 F). At these temperatures, water, methane and carbon dioxide are all in gaseous form. It is also known to be losing hydrogen gas at a ferocious rate, which makes the planet resemble a comet! The planet itself has a mass that is 69% that of Jupiter, and a volume that is 146% greater than that of Jupiter. The unofficial name for this planet is Osiris.

Problem 1 - The mass of Jupiter is 1.9×10^{30} grams. The radius of Jupiter is 71,500 kilometers. A) What is the volume of Jupiter in cubic centimeters, assuming it is a perfect sphere? B) What is the density of Jupiter in grams per cubic centimeter (cc), based on its mass and your calculated volume?

Problem 2 - From the information provided; A) What is the volume of Osiris in cubic centimeters, if it is in the shape of a perfect sphere? B) What is the mass of Osiris in grams? C) What is the density of Osiris in grams/cc, and how does this compare to the density of Jupiter?

Problem 3 - The densities of some common ingredients for planets are as follows:

Rock	3 grams/cc	Ice	1 gram/cc
Iron	9 grams/cc	Mixture of hydrogen + helium	0.7 grams/cc
Water	5 grams/cc		

Based on the average density of Osiris, from what substances do you think the planet is mostly composed?

Problem 1 - The mass of Jupiter is 1.9×10^{30} grams. The radius of Jupiter is 71,500 kilometers.
A) What is the volume of Jupiter in cubic centimeters, assuming it is a perfect sphere?
Answer: The radius of Jupiter, in centimeters, is
R = 71,500 km x (100,000 cm/1 km)
$= 7.15 \times 10^{9}$ cm.

For a sphere, $V = 4/3\, \pi\, R^3$ so the volume of Jupiter is
$V = 1.33 \times (3.141) \times (7.15 \times 10^{9})^3$
$V = 1.53 \times 10^{30}\ cm^3$

B) What is the density of Jupiter in grams per cubic centimeter (cc), based on its mass and your calculated volume?
Answer: Density = Mass/Volume so the density of Jupiter is D = (1.9×10^{30} grams)/($1.53 \times 10^{30}\ cm^3$) = 1.2 gm/cc

Problem 2 - From the information provided;
A) What is the volume of Osiris in cubic centimeters, if it is in the shape of a perfect sphere?
Answer: The information says that the volume is 146% greater than Jupiter so it will be V =
$V=1.46 \times (1.53 \times 10^{30}\ cm^3)$
$= 2.23 \times 10^{30}\ cm^3$

B) What is the mass of Osiris in grams?
Answer: the information says that it is 69% of Jupiter so
M = 0.69 x (1.9×10^{30} grams)
$= 1.3 \times 10^{30}$ grams
C) What is the density of Osiris in grams/cc, and how does this compare to the density of Jupiter?
Answer: D = Mass/volume
= 1.3×10^{30} grams / $2.23 \times 10^{30}\ cm^3$
= 0.58 grams/cc

Problem 3 - The densities of some common ingredients for planets are as follows:

Rock	3 grams/cc	Ice	1 gram/cc
Iron	9 grams/cc	Mixture of hydrogen + helium	0.7 grams/cc
Water	5 grams/cc		

Based on the average density of Osiris, from what substances do you think the planet is mostly composed?

Answer: Because the density of Osiris is only about 0.6 grams/cc, the closest match would be a mixture of hydrogen and helium. This means that, rather than a solid planet like earth, which is a mixture of higher-density materials such as iron, rock and water, Osiris has much in common with Jupiter which is classified by astronomers as a Gas Giant!

Artist's rendition of GJ 1214b with a hypothetical moon.
(Courtesy: CfA/David Aguilar)

In December 2009, astronomers announced the discovery of the transiting super-Earth planet GJ 1214b located 42 light years from the sun, and orbits a red-dwarf star. Careful studies of this planet, which orbits a mere 2 million km from its star and takes 1.58 days to complete 'one year'. Its mass is known to be 6.5 times our Earth and a radius of about 2.7 times Earth's. Its day-side surface temperature is estimated to be 370 F, and it is locked so that only one side permanently faces its star.

When a planet passes in front of its star, light from the star passes through any atmosphere the planet might contain and travels onwards to reach Earth observers. Although the disk of the planet will temporarily decrease the brightness of its star by a few percent, the addition of an atmosphere causes an additional brightness decrease. The amount depends on the thickness of the atmosphere, the presence of dust and clouds, and the chemical composition. By studying the light dimming at many different wavelengths, astronomers can distinguish between different atmospheric constituents by using specific spectral 'fingerprints'. They can also estimate the thickness of the atmosphere in relation to the diameter of the planet.

Problem 1 – Assuming the planet is a sphere, from the available information, to two significant figures, what is the average density of the planet in kg/meter^3? (Earth mass = 6.0×10^{24} kg; Diameter = 6378 km).

Problem 2 – The average density of Earth is 5,500 kg/m^3. Suppose that GJ 1214b has a rocky core with Earth's density and a radius of R, and a thin atmosphere with a density of D. Let R = 1.0 at the surface of the planet and R=0 at the center, and assume the core is a sphere, and that the atmosphere is a spherical shell with inner radius R and outer radius R=1.0. The formula relating the atmosphere density, D, to the core radius, R, is given by:

$$1900 = (5500 - D)R^3 + D$$

A) Re-write this equation by solving for D.

B) Graph the function D(R) over the domain R:[0,1].

C) If the average density of the atmosphere is comparable to that of Venus's atmosphere for which D= 100 kg/m^3, what fraction of the radius of the planet is occupied by the Earth-like core, and what fraction is occupied by the atmosphere?

Problem 1 – Answer: Volume = 4/3 (3.141) $(2.7 \times 6378000)^3$ = 2.1 x 10^{22} meter^3.
Density = Mass/Volume
= (6.5x6.0 x 10^{24} kg) / (2.1 x 10^{22} meter^3)
= **1,900 kg/meter^3**

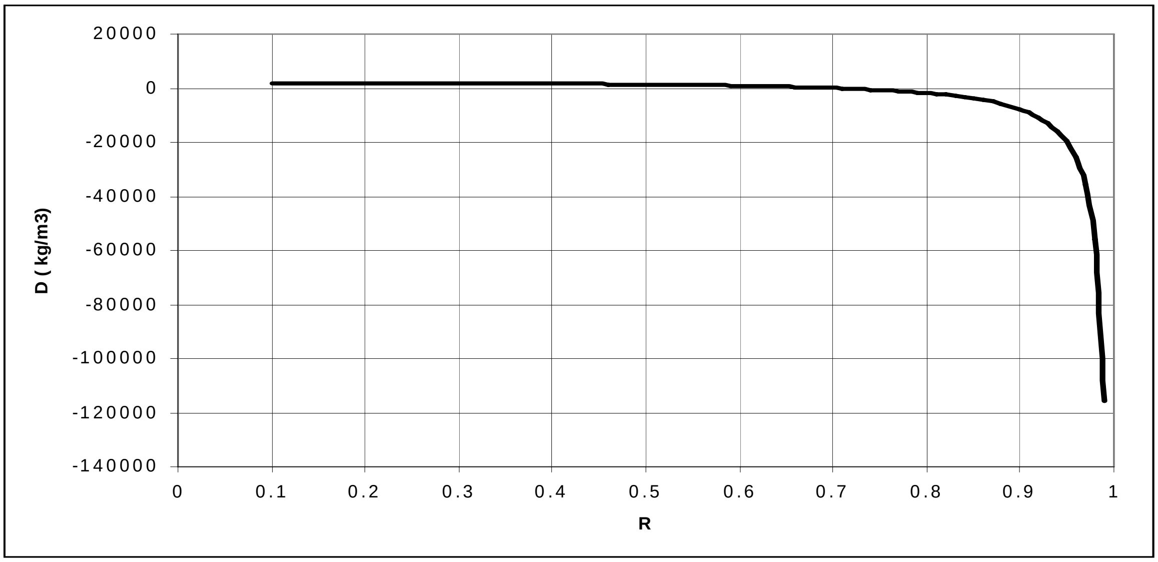
Problem 2 – A) Answer: $1900 = (5500 - D)R^3 + D$

$$1900 - D = 5500R^3 - DR^3$$

$$D\left(R^3 - 1\right) = 5500R^3 - 1900$$

$$D = \frac{5500R^3 - 1900}{R^3 - 1}$$

B) See below:



Note that for very thin atmospheres where D: [0, 100] the function predicts that the core has a radius of about R=0.7 or 70% of the radius of the planet. Since the planet's radius is 2.7 times earth's radius, the core is about 0.7x2.7 = 1.9 x earth's radius. Values for D < 0 are unphysical even though the function predicts numerical values. This is a good opportunity to discuss the limits of mathematical modeling for physical phenomena.

C) If the average density of the atmosphere is comparable to that of Venus's atmosphere for which D= 100 kg/m^3, what fraction of the radius of the planet is occupied by the Earth-like core, and what fraction is occupied by the atmosphere?

Answer: **For D=100, R = 0.71, so the core occupies the inner 71 % of the planet, and the surrounding atmospheric shell occupies the outer 29% of the planet's radius.**

A very common way to describe the atmosphere of a planet is by its 'scale height'. This quantity represents the vertical distance above the surface at which the density or pressure if the atmosphere decreases by exactly 1/e or $(2.718)^{-1}$ times (equal to 0.368).

The scale height, usually represented by the variable **H**, depends on the strength of the planet's gravity field, the temperature of the gases in the atmosphere, and the masses of the individual atoms in the atmosphere. The equation to the left shows how all of these factors are related in a simple atmosphere model for the density P. The variables are:

z: Vertical altitude in meters
T: Temperature in Kelvin degrees
m: Average mass of atoms in kilograms
g: Acceleration of gravity in meters/sec^2
k: Boltzmann's Constant 1.38×10^{-23} J/deg

$$P(z) = P_0 \ e^{-\frac{z}{H}} \quad \text{and} \quad H = \frac{kT}{mg}$$

Problem 1 - For Earth, g = 9.81 meters/sec^2, T = 290 K. The atmosphere consists of 22% O_2 (m= $2\times2.67\times10^{-26}$ kg) and 78% N_2 (m= $2\times2.3\times10^{-26}$ kg). What is the scale height, H?

Problem 2 - The thickness of a planetary atmosphere is not very well defined, although the scale height for the atmosphere can be calculated as we have just seen. One definition might be the altitude which encloses 99% of the planet's total mass defined by the formula $1/100 = e^{-h/H}$. Solve this formula for h given that the scale height, H, for Earth's atmosphere is 8.6 km.

Problem 3 - Another way to define the thickness of a planetary atmosphere is the altitude of the last, stable, satellite orbit. This is tricky because atmospheric friction causes satellite orbits to decay within an amount of time that varies from thousands of years to hours, depending on the density of the air. For Earth, a satellite with an altitude of 150 kilometers will decay in a time less than its orbit period. For a scale height of 8.6 kilometers, what is the density of the atmosphere at this altitude compared to sea-level?

Problem 1 - Answer: First we have to calculate the average atomic mass. <m> = 0.22 ($2 \times 2.67 \times 10^{-26}$ kg) + 0.78 ($2 \times 2.3 \times 10^{-26}$ kg) = 4.76×10^{-26} kg. Then,

$$H = \frac{(1.38 \times 10^{-23})(290)}{(4.76 \times 10^{-26})(9.81)} =$$ **8,570 meters or about 8.6 kilometers.**

Problem 2 - The thickness of a planetary atmosphere is not very well defined, although the scale height for the atmosphere can be calculated as we have just seen. One definition might be the altitude which encloses 99% of the planet's total mass defined by the formula $1/100 = e^{-h/H}$. Solve this formula for h given that the scale height, H, for Earth's atmosphere is 8.6 km.

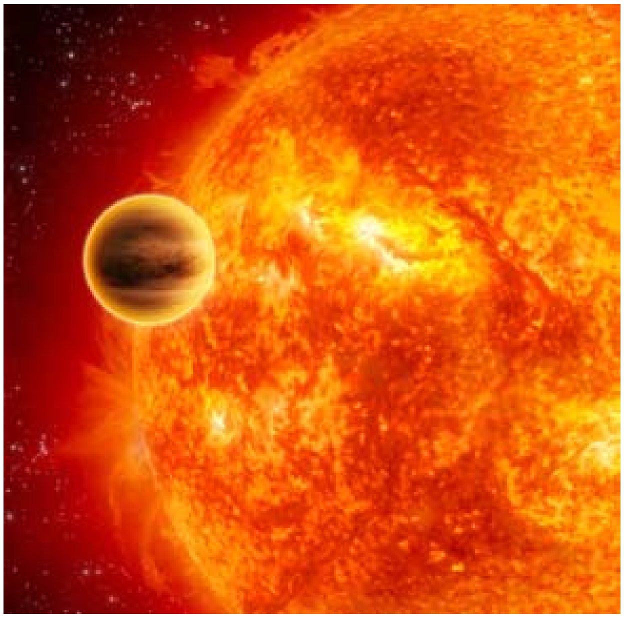
Answer: $100 = e^{-(h/8.6)}$. Taking the natural-log ln of both sides we get ln(100) = h/8.6 so h = 8.6 ln(100) and **h = 39.6 km.**

Problem 3 - Another way to define the thickness of a planetary atmosphere is the altitude of the last, stable, satellite orbit. This is tricky because atmospheric friction causes satellite orbits to decay within an amount of time that varies from thousands of years to hours, depending on the density of the air. For Earth, a satellite with an altitude of 150 kilometers will decay in a time less than its orbit period. For a scale height of 8.6 kilometers, what is the density of the atmosphere, N, at this altitude compared to sea-level, n?

Answer: $N = n\, e^{-(150km/8.6km)}$

$N/n = 2.7 \times 10^{-8}$

In other words, the atmosphere is about 1/270 million times less dense!



Over 430 planets have been discovered orbiting nearby stars since 1995. Called 'exoplanets' to distinguish them from the familiar 8 planets in our own solar system, they are planets similar to Jupiter in size, but orbiting their stars in mostly elliptical paths. In many cases, the planets come so close to their star that conditions for life to exist would be impossible.

Astronomers are continuing to search for smaller planets to find those that are more like our own Earth.

(Artist rendition: courtesy NASA)

Use the basic properties and formulae for ellipses to analyze the following approximate exoplanet orbits by first converting the indicated equations into standard form. Then determine for each planet the:

A) a= semi-major axis

B) b=semi-minor axis;

C) ellipticity $e = \frac{\sqrt{(a^2 - b^2)}}{a}$

D) 'perihelion' closest distance to star, defined as P=a(1-e);

E) 'aphelion' farthest distance from star, defined as A = a(1+e).

Problem 1: Planet: 61 Virginis-d Period=4 days $1 = 4x^2 + 5y^2$

Problem 2: Planet: HD100777-b Period=383 days $98 = 92x^2 + 106y^2$

Problem 3: Planet: HD 106252-b Period=1500 days $35 = 5x^2 + 7y^2$

Problem 4: Planet: 47 UMa-c Period= 2190 days $132 = 11x^2 + 12y^2$

Problem 1: 61 Virginis -d Period=4 days $1 = 4x^2 + 5y^2$

$$1 = \frac{x^2}{0.25} + \frac{y^2}{0.20}$$

a=0.5 b=0.45 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.43** P = (0.5)(1-0.43) = **0.28**, A=(0.5)(1+0.43) = **0.71**

Problem 2: Planet: HD100777-b Period=383 days $98 = 92x^2 + 106y^2$

$$1 = \frac{x^2}{1.06} + \frac{y^2}{0.92}$$

a=1.03 b=0.96 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.36** P = (1.03)(1-0.36) = **0.66,** A=(1.03)(1+0.36) = **1.40**

Problem 3: Planet: HD 106252-b Period=1500 days $35 = 5x^2 + 7y^2$

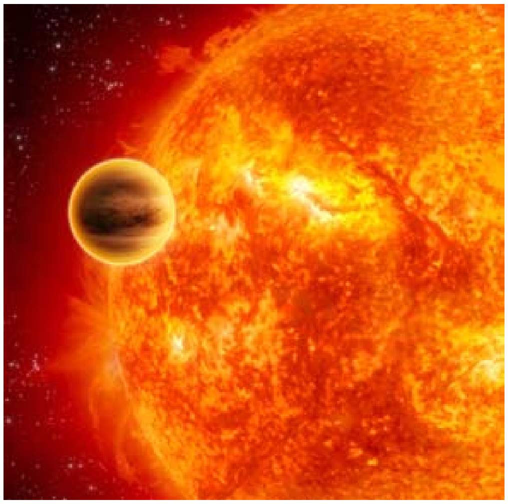
$$1 = \frac{x^2}{7.0} + \frac{y^2}{5.0}$$

a=2.6 b=2.2 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.53** P = (2.6)(1-0.53) = **1.22**, A=(2.6)(1+0.53) = **4.0**

Problem 4: Planet: 47 UMa-c Period= 2190 days $132 = 11x^2 + 12y^2$

$$1 = \frac{x^2}{12.0} + \frac{y^2}{11.0}$$

a=3.5 b=3.3 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.33** P = (3.5)(1-0.33) = **2.35,** A=(3.5)(1+0.33) = **4.65**



Because many exoplanets orbit their stars in elliptical paths, they experience large swings in temperature. Generally, organisms can not survive if water is frozen (0 C = 273 K) or near its boiling point (100 C or 373 K). Due to orbital conditions, this very narrow 'zone of life' may not be possible for many of the worlds detected so far.

Problem 1 - Complete the table below by calculating
A) The semi-minor axis distance $B = A(1-e^2)$
B) The perihelion distance $Dp = A(1-e)$
C) The aphelion distance, $Da = B(1+e)$

Problem 2 - Write the equation for the orbit of 61 Vir-d in Standard Form.

Problem 3 - The temperature of a planet similar to Jupiter can be approximated by the formula below, where T is the temperature in Kelvin degrees, and R is the distance to its star in AU. Complete the table entries for the estimated temperature of each planet at the farthest 'aphelion' distance Ta, and the closest 'perihelion;' distance Tp.

$$T(R) = \frac{250}{\sqrt{R}}$$

Problem 4 - Which planets would offer the most hospitable, or the most hazardous, conditions for life to exist, and what would be the conditions be like during a complete 'year' for each world?

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39		0.22	2190				
61 Vir-d	0.47		0.35	123				
HD106252-b	2.61		0.54	1500				
HD100777-b	1.03		0.36	383				
HAT-P13c	1.2		0.70	428				

Problem 1 See table below:

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39	**3.3**	0.22	2190	**2.6**	**4.1**		
61 Vir-d	0.47	**0.4**	0.35	123	**0.3**	**0.6**		
HD106252-b	2.61	**2.2**	0.54	1500	**1.2**	**4.0**		
HD100777-b	1.03	**1.0**	0.36	383	**0.7**	**1.4**		
HAT-P13c	1.2	**0.9**	0.70	428	**0.4**	**2.0**		

Problem 2 Write the equation for the orbit of 61 Vir-d in Standard Form.
Answer: A = 0.47 and B = 0.4

So $1 = \frac{x^2}{0.47} + \frac{y^2}{0.40}$ and also $188 = 40x^2 + 47y^2$

Problem 3 - See table below:

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39	**3.3**	0.22	2190	**2.6**	**4.1**	**154**	**123**
61 Vir-d	0.47	**0.4**	0.35	123	**0.3**	**0.6**	**452**	**314**
HD106252-b	2.61	**2.2**	0.54	1500	**1.2**	**4.0**	**228**	**125**
HD100777-b	1.03	**1.0**	0.36	383	**0.7**	**1.4**	**308**	**211**
HAT-P13c	1.2	**0.9**	0.70	428	**0.4**	**2.0**	**417**	**175**

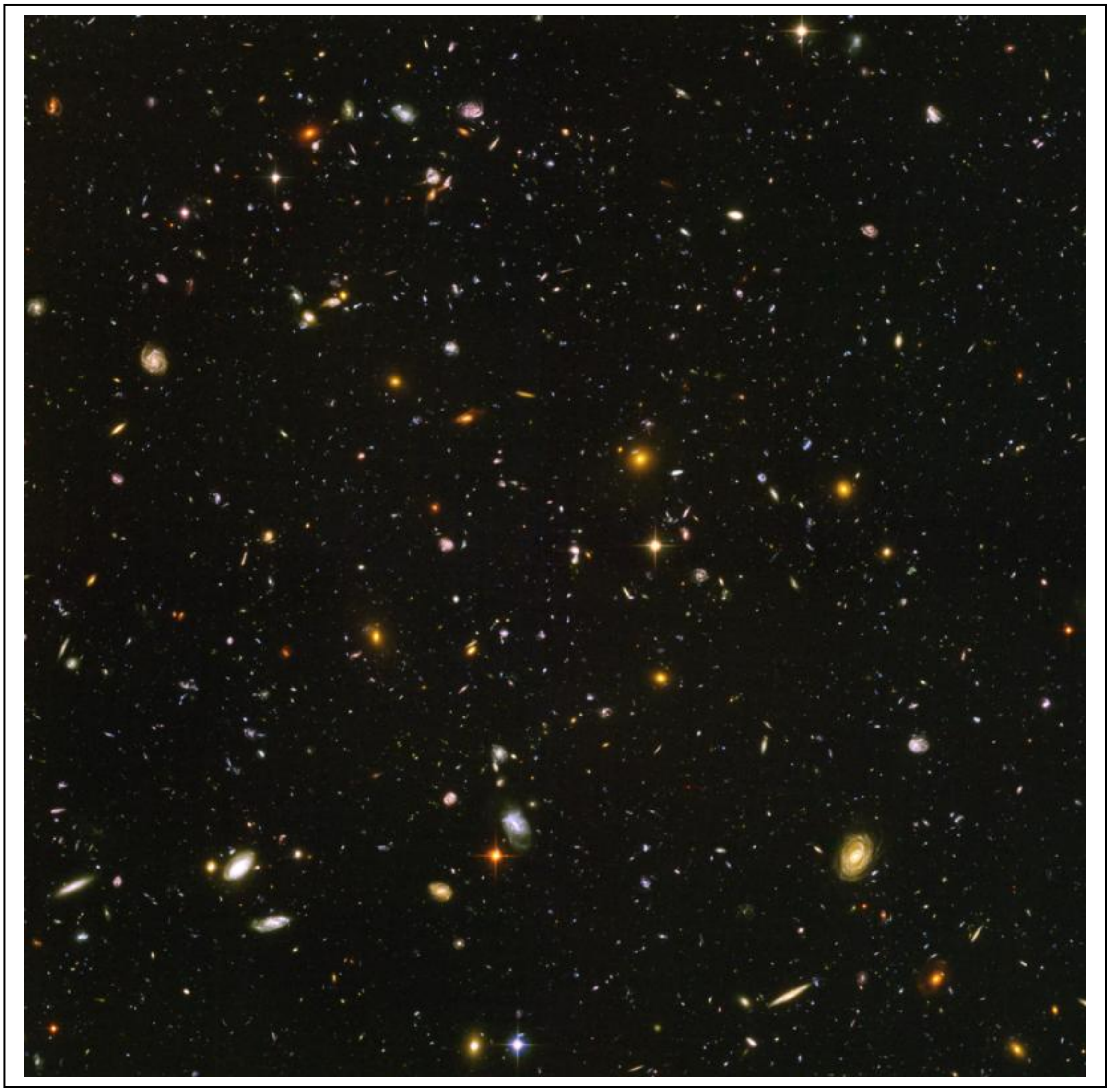
Problem 4 - Which planets would offer the most hospitable, or most hazardous, conditions for life to exist, and what would be the conditions be like during a complete 'year' for each world?
Answer: For the habitable 'water' range between 273K and 373K, none of these planets satisfy this minimum and maximum condition. They are either too hot at perihelion 'summer' or too cold at 'winter' aphelion.

Only HD100777-b during perihelion is in this temperature range during 'summer', at a temperature of 308 K (35 C). During 'winter' at aphelion, it is at -62 C which is below the freezing point of water, and similar to the most extreme temps in Antarctica.

Note: These temperature calculations are only approximate and may be considerably different with greenhouse heating by the planetary atmosphere included.

Counting Galaxies with the Hubble Space Telescope

In 2004, the Hubble Space Telescope took a million-second exposure of a small part of the sky to detect as many galaxies as possible. Here's what they saw!



Problem 1 – Divide the field into 16 equal areas. Label the grids alphabetically from A to P starting from the top left cell. Count the number of 'smudges' in four randomly selected cells. What is the average number of galaxies in of one of the cells in the picture? What uncertainties can you identify in counting these galaxies?

Problem 2 – One square degree equals 3,600 square arcminutes. If Hubble Ultra Deep Field picture is 3 arcminutes on a side, what is the area of one of your cells in square degrees?

Problem 3 – There are 41,250 square degrees in the sky, about how many galaxies are in the full sky as faint as the faintest galaxy that Hubble detected in the Deep Field image?

Answer Key

Problem 1 – Divide the field into 16 equal areas. Count the number of 'smudges' in four randomly selected cells. What is the average number of galaxies in an area of the sky equal to the size of one of the cells in the picture? What uncertainties can you identify in counting these galaxies?

Answer: Students counting will vary depending on how many 'spots' they can easily see. There should be vigorous discussions about which smudges are galaxies and which ones are photocopying artifacts. Note that, depending on the quality of the laser printer used, the number of galaxies will vary considerably.

The full image is about 145 millimeters wide, so the cells should measure about 145/4 = 36 millimeters wide. For a typical photocopy quality, here are some typical counts in 4 cells:
D=160, F=170; K=112; M=164. The average is (160+170+112+164)/4 = 151 galaxies/cell. Students estimates may vary depending on the number of galaxies they could discern in the photocopy of the Hubble Ultra-Deep Field image. A reasonable range is from 50 – 200 galaxies per cell on average.

Problem 2 – One square degree equals 3,600 square arcminutes. If the Hubble Ultra Deep Field picture is 3 arcminutes on a side, what is the area of one of your cells in square degrees?

Answer: The field is 3 arcminutes wide, so one cell is 3 / 4 = 0.75 arcminutes on a side. Since there are 60 arcminutes in one degree, this equals 0.75 x 1/60 = 0.0125 degrees. The area of the cell is 0.0125 x 0.0125 = 0.000156 square degrees.

Problem 3 – There are 41,250 square degrees in the sky, about how many galaxies are in the full sky as faint as the faintest galaxy that Hubble detected in the Deep Field image?

Answer: The number of these cells in the full sky is 41,250 square degrees/0.000156 square degrees = 264 million. Since there are on average 151 galaxies per cell, the total number of galaxies in the sky is 151 galaxies/cell x 264 million cells/full sky = 39.4 billion galaxies.

Note, astronomers using the original image data counted an average of 625 galaxies in each cell, for an estimated total of 165 billion galaxies in the full sky.

Galaxies to Scale

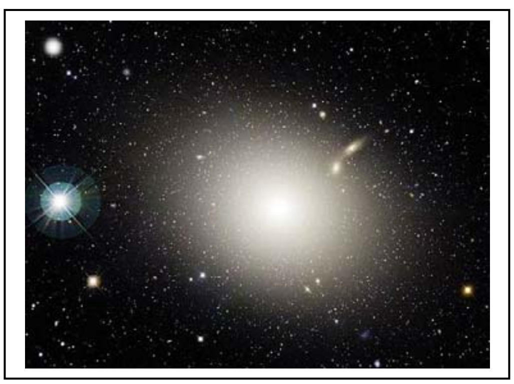
The Milky Way is a spiral galaxy. There are many other kinds of galaxies, some much larger then the Milky Way, and some much smaller. This exercise lets you create a scale model of the various kinds, and learn a little about working with fractions too!

Problem 1 - The irregular galaxy IC-1613 is twice as large as the elliptical galaxy M-32, but 10 times smaller than the spiral galaxy NGC-4565. How much larger is NGC-4565 than M-32?

Problem 2 - The spiral galaxy Andromeda is three times as large as the elliptical galaxy NGC-5128, and NGC-5128 is 4 times as large as the Large Magellanic Cloud, which is an irregular galaxy. How much larger is the Andromeda galaxy than the Large Magellanic Cloud?

Problem 3 - The Milky Way spiral galaxy is 13 times larger than the irregular galaxy IC-1613. How much larger than NGC-4565 is the Milky Way?

Problem 4 - The elliptical galaxy Leo-1 is 1/4 as large as the elliptical galaxy Messier-32, and the spiral galaxy Messier-33 is 9 times larger than Messier-32. How large is Leo-1 compared to Messier-33?



Problem 5 - The elliptical galaxy NGC-205 is 2/3 as large as the Large Magellanic Cloud. How large is NGC-205 compared to the Andromeda galaxy?

Problem 6 - The irregular galaxy NGC-6822 is 8/5 the diameter of Messier-32, and Messier-32 is 20 times smaller than NGC-4565. How large is NGC-6822 compared to IC-1613?

Problem 7 - Draw a scale model of these galaxies showing their relative sizes and their shapes.

Images: Top: The spiral galaxy Messier 74 taken by the Gemini Observatory; Bottom: The elliptical galaxy Messier-87 obtained at the Canada-France-Hawaii Telescope (copyright@cfht.hawaii.edu);

Answer Key

The galaxies used in this exercise, with the diameter given in light years, and relative to Messier-32:

Name	Type	Diameter	M-32
Large Magellanic Cloud	Irregular	15,000 LY	3
NGC-5128	Elliptical	65,000	13
NGC-4565	Spiral	100,000	20
IC-1613	Irregular	10,000	2
Andromeda	Spiral	200,000	40
NGC-205	Elliptical	10,000	2
Messier-32	Elliptical	5,000	1
Milky Way	Spiral	130,000	26
Messier-33	Spiral	45,000	9
Leo-1	Elliptical	1,000	1/4
NGC-6822	Irregular	8,000	8/5

Problem 1 - IC-1613/M-32 = 2.0 , NGC-4565/IC-1613 = 10 so NGC-4565/M-32 = 10 x 2 = **20 times**

Problem 2 - Andromeda/NGC-5128 = 3 and NGC-5128/LMC = 4 so Andromeda/LMC = 3 x 4 =**12 times**

Problem 3 - MW/IC-1613 = 13 and also NGC-4565/IC-1613 = 10, so Milky Way / NGC-4565 = 13 x 1/10 = **1.3 times.**

Problem 4 - Leo-1 / M-32 = 1/4 and M-33 / M-32 = 9, so Leo-1 / M-33 = 1/4 x 1/9 = **1/36 times smaller**.

Problem 5 - NGC-205 / LMC = 2/3 and Andromeda/LMC = 12 so NGC-205 / Andromeda = 2/3 x 1/12 = **2/36 or 1/18 as large.**

Problem 6 - NGC-6822 / M-32 = 8/5 and M-32 / NGC-4565 = 1/20 and NGC-4565 / IC-1613 = 10 so NGC-6822 / IC-1613 = 8/5 x 1/20 x 10 = 8/5 x 1/2 = **8/10 or 4/5 as large.**

Problem 7 - Students can use the ratios in the problems, together with the ones they derived, to create a table that gives the relative sizes for each galaxy. The table at the top gives the 'official' numbers, and the relative sizes in the last column.

Galaxy Distances and Mixed Fractions

Our Milky Way galaxy is not alone in the universe, but has many neighbors.

The distances between galaxies in the universe are so large that astronomers use the unit 'megaparsec' (mpc) to describe distances.

One *mpc* is about 3 1/4 million light years.

Hubble picture of a Ring Galaxy (AM 0644 741) at a distance of 92 mpc.

Problem 1 - The Andromeda Galaxy is 3/4 mpc from the Milky Way, while the Sombrero Galaxy is 12 mpc from the Milky Way. How much further is the Sombrero Galaxy from the Milky Way?

Problem 2 -The Pinwheel Galaxy is 3 4/5 mpc from the Milky Way. How far is it from the Sombrero Galaxy?

Problem 3 - The Virgo Galaxy Cluster is 19 mpc from the Milky Way. About how far is it from the Pinwheel Galaxy?

Problem 4 - The galaxy Messier 81 is located 3 1/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy?

Problem 5 - The galaxy Centaurus-A is 4 2/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy?

Problem 6 - The galaxy Messier 63 is located about 4 1/5 mpc from the Milky Way. How far is it from the Pinwheel galaxy?

Problem 7 - The galaxy NGC-55 is located 2 1/3 mpc from the Milky Way. How far is it from the Andromeda galaxy?

Problem 8 - In the previous problems, which galaxy is 2 1/15 mpc further from the Milky Way than NGC-55?

Extra for Experts: How far, in light years, is the Virgo Galaxy Cluster from the Milky Way?

Answer Key

Problem 1 - The Andromeda Galaxy is 3/4 mpc from the Milky Way, while the Sombrero Galaxy is 12 mpc from the Milky Way. How much further is the Sombrero Galaxy from the Milky Way? Answer: 12 mpc - 3/4 mpc = **11 1/4 mpc**

Problem 2 -The Pinwheel Galaxy is 3 4/5 mpc from the Milky Way. How far is it from the Sombrero Galaxy? Answer: 12 mpc - 3 4/5 mpc = **8 1/5 mpc**

Problem 3 - The Virgo Galaxy Cluster is 19 mpc from the Milky Way. About how far is it from the Pinwheel Galaxy? Answer: 19 mpc - 3 4/5 mpc = **15 1/5 mpc.**

Problem 4 - The galaxy Messier 81 is located 3 1/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy? Answer: 3 1/5 mpc - 3/4 mpc = 16/5 mpc - 3/4 mpc = 64/20 mpc - 15/20 mpc = 49/20 mpc = **2 9/20 mpc.**

Problem 5 - The galaxy Centaurus-A is 4 2/5 mpc from the Milky Way. How far is it from the Andromeda Galaxy? Answer: 4 2/5 mpc - 3/4 mpc = 88/5 mpc - 15/20 mpc = 73/20 mpc = **3 13/20 mpc**

Problem 6 - The galaxy Messier 63 is located about 4 1/5 mpc from the Milky Way. How far is it from the Pinwheel galaxy? Answer: 4 1/5 mpc - 3 4/5 mpc = 21/5 mpc - 19/5 mpc = **2/5 mpc.**

Problem 7 - The galaxy NGC-55 is located 2 1/3 mpc from the Milky Way. How far is it from the Andromeda galaxy? Answer: 2 1/3 mpc - 3/4 mpc = 7/3 mpc - 3/4 mpc = 28/12 mpc - 9/12 mpc = 19/12 mpc = **1 7/12 mpc**.

Problem 8 - In the previous problems, which galaxy is 2 1/15 mpc further from the Milky Way than NGC-55? Answer; NGC-55 is located 2 1/3 mpc from the Milky Way, so the mystery galaxy is located 2 1/3 mpc + 2 1/15 mpc = 7/3 mpc + 31/15 mpc = 35/15 mpc + 31/15 mpc = 66/15 mpc = 4 6/15 mpc or **4 2/5 mpc**. This is the distance to the Centaurus-A galaxy.

Extra for Experts: How far, in light years, is the Virgo Galaxy Cluster from the Milky Way?
Answer: The distance is 19 megaparsecs, but 1 parsec equals 3 1/4 light years, so the distance to the Virgo Cluster is

19 million parsecs x (3 1/4 lightyears/parsec) = 19 x 3 1/4 = 19 x 12/4 = 228/4 = **57 million light years**

Note: Galaxies are actually located in 3-dimensional space, but to make this problem work we have assumed that the galaxies are all located along a straight like with the Milky Way at the center.

Exploring a Galaxy : NGC-1232

The galaxy NGC-1232 was photographed by the ESA, Very Large Telescope in May, 2003. From resources at your library or the Internet, fill-in the following information:

1) Type of galaxy ---------- Number of arms ----------
2) Name of galaxy cluster it is a member of --------
3) Distance in parsecs and in light-years ----------
4) Right Ascension ------- Declination------
5) Constellation ---------
6) Diameter in light years ------
7) Diameter in arcminutes -----
8) Apparent visual magnitude ----------

From the photograph below, and an assumed diameter of 200,000 light years, answer these questions:

9) Diameter of nuclear region in light years ---------------
10) Average width of arms in light years ----------
11) Average spacing between arms in light years--------
12) Diameter of brightest star clusters (bright knots) in light years ------------
13) Describe in 500 words why this galaxy is so interesting.

1) Type of galaxy ---------- Spiral 'Sc' Number of arms ---------- About 5
2) Name of galaxy cluster it is a member of -------- Eridanus Galaxy Group
3) Distance in light-years ---------- 70 to 100 million Light-years
This question introduces students to the many different values that can be cited, especially on the internet. Earlier estimates (70 million) sometimes are given even though more recent estimates (100 million) are available. For research purposes, astronomers will always use the most current estimate and will state why they do so.
4) Right Ascension ------- 3h 9m 45s Declination------ -20d 34'
5) Constellation ----- Eridanus
6) Diameter in light years ------ 100,000 light years (or 200,000 lightyears)
The diameter is based on the angular sized (which is fixed) and the distance (which depends on whether you use the 70 or 100 million light year estimate). At a distance of 100 million light years, the diameter is about 200,000 light years. At 70 million light years, the diameter is (70/100)*200,000 = 140,000 light years. Students may find articles where authors cite either 100,000 or 200,000 light years. This might be a good time to ask students which resources preferred one estimate over another, and why there is such uncertainty. How do astronomers measure the distances to galaxies?
7) Diameter in arcminutes ----- 7 arc minutes (moon is 30 arcminutes!)
This is the diameter of the galaxy as you would see it in the sky from Earth. The moon is 30 arcminutes in diameter, so NGC-1232 is about ¼ the diameter of the full moon. Students may need to be reminded that there are 360 degrees in a full circle, and each degree consists of 60 minutes of arc.
8) Apparent visual magnitude ---------- +10.6
This is a measure of how bright the galaxy appears in the sky. The faintest stars you can see in a rural dark sky are about +6, while in an urban setting this limit is about +3. Because each magnitude corresponds to a brightness factor of 2.5, the galaxy is about 8 magnitudes fainter than the brightest star you can see in a city, or a factor of 2.5 x 2.5 x 2.5 x 2.5 x 2.5 x 2.5 x 2.5 x 2.5 = 1500 fainter.

From the photograph, and an assumed diameter of 200,000 light years, answer these questions:
The diameter of the galaxy photograph is about 100 millimeters, so the scale of the photograph is 200,000 light years / 100 mm = 2,000 light years per millimeter.

9) Diameter of nuclear region. About 10 millimeters or 20,000 light years.
10) Average width of arms. Students will find that the arm widths near the nucleus are narrower than in the more distant regions. They vary from about 4mm (8,000 light years) to 10 mm (20,000 light years).
11) Average spacing between arms. As for the arm widths, this can vary from 4mm (8,000 light years) to 25 mm (50,000 light years).
12) Diameter of brightest star clusters (bright knots). About 2 mm (4,000 light years).
13) Describe in 500 words why this galaxy is so interesting. Students may cite many features, including its similarity to the Milky Way, the complexity of its arms, the many star clusters, the complexity and shape of the nuclear region.

Star clusters up close

This is the Pleiades star cluster. From resources at your library or the Internet, fill-in the following information:

1) Type of cluster ---------- Number of stars ----------
2) Alternate Names -----
3) Distance in light-years ----------
4) Right Ascension ------- Declination------
5) Constellation ---------
6) Diameter in light years ------
7) Diameter in arcminutes -----
8) Apparent visual magnitude ----------
9) How old is the star cluster? -----------
10) What kinds of stars can you find in the cluster? ------
11) What are some of the names of the stars? --------

From the photograph below, and the cluster's diameter light years, answer these questions:

12) How many stars are probably members of the cluster? --------
13) What is the average distance between the brightest stars? -----
14) What is the typical distance between the stars you counted in question 12? ------
15) Why do the stars have spikes? --------
16) Describe in 500 words why this star cluster is so interesting.

1) Type of cluster ---------- **Open, Galactic Cluster** Number of stars ----------**500**
2) Alternate Names -------- **Seven Sisters; Messier-45**
3) Distance in light-years ---------- **425 light years.**
4) Right Ascension ------- **3h 47m** Declination------ **+24d 00'**
5) Constellation --------- **Taurus**
6) Diameter in light years ------ **About 12 light years.**
7) Diameter in arcminutes ----- **110' or about 3 times the diameter of the full moon!**
8) Apparent visual magnitude ---------- **Between +5 and +6.**
9) How old is the star cluster? ----------- **About 100 million years.**
10) What kinds of stars can you find in the cluster? – **Mostly type-B main sequence**
11) What are some of the names of the stars? **Pleione, Atlas, Merope, Alcyone, Electra, Maia, Asterope, Taygeta, Celeano.**

From the photograph below, and the cluster's diameter light years, the size of the cluster is about 120 mm which equals 12 light years for a scale of 10 millimeters to one light year.

12) How many stars in the photo are probably members of the cluster?
Students should notice that there is a distinct gap between the bright stars and the faint stars in the photo. The faint stars are mostly background stars in the Milky Way unrelated to the cluster. By squinting at the photo, students should be able to find about 100-120 stars.
13) What is the average distance between the brightest stars? ----- **The seven bright stars are about 25 mm or 2.5 light years apart.**
14) What is the typical distance between the stars you counted in question 12? – **With a millimeter ruler, students can measure the spaces between a few dozen stars in the picture and find an average, or they can squint at the picture and use their ruler to estimate the answer. Answers between 5 and 10 mm are acceptable and equal 0.5 to 1 light year.**
15) Why do the stars have spikes? -- **Students may need to investigate this question by using books or the web. Generally, reflecting telescopes produce stellar spikes because the secondary mirror diffracts some of the starlight. The spikes are the four 'legs' used to support the smaller mirror inside the telescope. Refractors do not have spikes and produce round images. In no case do the round images suggest that the star is actually being resolved.**

16) Describe in 500 words why this star cluster is so interesting.

Students will find many items on the web to form the basis for their essay including: The Pleiades cluster has a long history in mythology. There are many names for this cluster throughout many civilizations and languages. Astronomically, it is the closest open cluster to the sun. It is very young, and contains many stars that are 100 to 1000 times more luminous than the sun. This cluster will eventually fade away in about 250 million years as its brightest stars evolve and die. The cluster still contains the gas left over from its formation, which can be seen as the nebula surrounding the six brightest stars.

Our Neighborhood in the Milky Way

The Milky Way galaxy is a flat disk about 100,000 light years in diameter and 1000 light years thick. All of the bright stars, clusters and nebula we see are actually very close-by. Let's have a look at a few of these familiar landmarks!

The table below gives the distances and angles to a few familiar nebulae and star clusters within 7,000 light years of the Sun. Plot them on a paper with a scale of 1 centimeter = 500 light years, and with the Sun at the origin.

Object	Type	Distance		Angle
Pleiades	star cluster	410	ly	60
Orion Nebula	nebula	1500		80
Betelgeuse	star	650		90
Deneb	star	1600		310
Antares	star	420		245
Cygnus Loop	supernova remnant	2000		315
Ring Nebula	nebula	2300		280
Owl Nebula	nebula	1900		170
Crab Nebula	supernova remnant	6300		80
Praesepe	star cluster	520		130
Rosette Nebula	nebula	3,600		100
Eta Carina	nebula	7,000		160
Lagoon Nebula	nebula	4,000		270
Jewel Box	star cluster	6,500		190

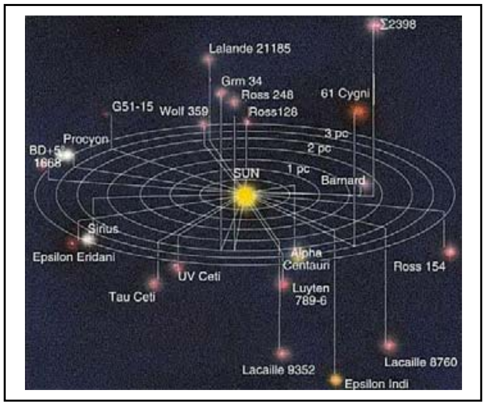
Problem 1 - If you only wanted to visit the three bright stars, how many light years would you have to travel for a round-trip tour?

Problem 2 - If you only wanted to visit all of the nebulas how long would your round-trip journey be?

Problem 1 - Sun - Betelgeuse - Deneb - Antares - sun would measure 10 centimeters, and from the scale of 1 cm=500 ly, it would be a **5,000 light year journey.**

Problem 2 - One possibility would be Sun - Cygnus Loop - Lagoon Nebula - Ring Nebula - Owl Nebula - Eta Carina Nebula - Rosette Nebula - Crab Nebula - Orion Nebula - Sun. This would be a journey of 45 centimeters or 45 x 500 = **22,500 light years.**

The Solar Neighborhood within 17 light years.



There are 45 stars within 17 light years of the sun. It is very hard to appreciate just how big space is. By considering our neighborhood in the Milky Way, we can start to get a sense of scale.

Local star map courtesy NASA/JPL

The table below gives the names, distances and angles to 11 of the most well-known neighbors to the sun. Although stars are spread out in 3-dimensional space, we will compress these distances to their 2-dimensional equivalents. On a 2-dimensional grid, place a dot at the Origin to represent the Sun. With a metric ruler and a protractor, plot the stars on a piece of paper and label each star. Use a scale of 1 centimeter = 1 light year.

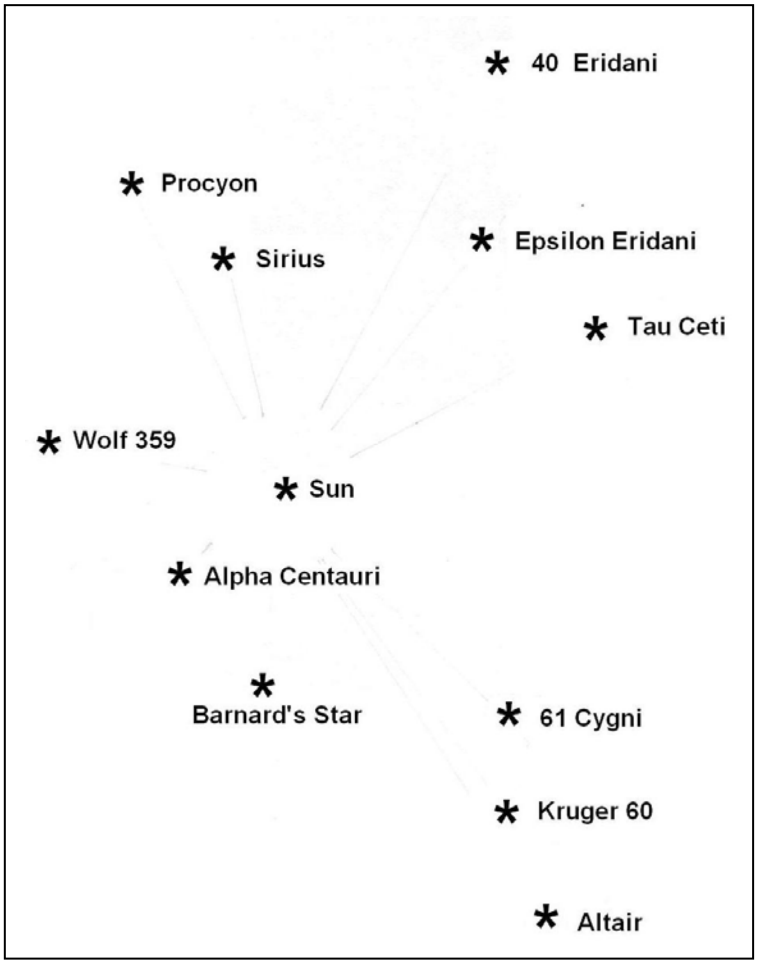
Name	Angle	Distance
Alpha Centauri	220	4.3 light years
Barnard's Star	270	5.9
Wolf 359	170	7.6
Sirius	100	8.6
Epsilon Eridani	50	10.7
61 Cygni	315	11.2
Procyon	115	11.4
Tau Ceti	25	11.9
Kruger 60	335	12.8
40 Eridani	60	15.9
Altair	300	16.6

Problem 1 - What is the distance between Sirius and Altair?

Problem 2 - What is the distance between Kruger 60 and Altair?

Problem 3 - Can you find a pair of stars that are closer to each other than either of them are to the Sun?

Problem 4 - If you were starting from Earth, what is the shortest journey you could make that would visit all of the stars in your map?



Problem 1 - At the scale of 1 cm = 1 light year, they are separated by about 24 centimeters or **24 light years.**

Problem 2 - They are separated by 7 centimeters or **7 light years.**

Problem 3 - For example: Procyon and Sirius are 4 cm apart or **4 light years.** 61 Cygni and Kruger 60 are 3 cm apart or **3 light years.**

Problem 4 - One path consists of Sun-Barnards Star-Altair-Kruger 60- 61 Cygni-Tau Ceti-Epsilon Eridani- 40 Eridani-Sirius-Procyon-Wolf 359-Alpha Centauri -Sun. The total distance is about **80 light years.**

How many stars are there?

On a clear night in the city you might be able to see a few hundred stars. In the country, far away from city lights, perhaps 5000 can be seen. Telescopes can see literally millions of stars. But how do we accurately count them? This exercise will show you the basic method!

This image was taken by the 2MASS sky survey. It is a field that measures 9.0 arcminutes on a side.

Problem 1 – By using a millimeter ruler, divide this star field into an equally-spaced grid that is 3 x 3 cells.

Problem 2 – Select 3 of these cells and count the number of star images you can see in each cell. Calculate the average number of stars in a cell.

Problem 3 – A square degree measures 60 arcminutes x 60 arcminutes in area. The full sky has an area of 41,253 square degrees. What are the total number of stars in A) one square degree of the sky; B) the number of stars in the entire sky.

Problem 4 – Why do you think we needed to average the numbers in Problem 2?

Answer Key

Problem 1 – By using a millimeter ruler, divide this star field into an equally-spaced grid that is 3 x 3 cells. Answer: An example is shown below.

Problem 2 – Select 3 of these cells and count the number of star images you can see in each cell. Calculate the average number of stars in a cell. Answer: Using the cells in the top row you may get: 159, 154 and 168. The average is 481/3 = 160 stars.

Problem 3 – Use the information in the text to convert your answer into the total number of stars in one square degree of the sky.

Answer: A) The answer from 3 is the number of stars in one cell. The area of that cell is 3 x 3 = 9 square arcminutes. One degree contains 60 arcminutes, so a square degree contains 60 x 60 = 3600 square arcminutes. Your estimated number of stars in one square degree is then 3600/9 = 40 times the number of stars you counted in one average one cell. For the answer to Problem 2, the number in a square degree would be 160 X 40 = 6,400 stars.

B) The text says that there are 41,253 square degrees in the full sky, so from your answer to Problem 3, you can convert this into the total number of stars in the sky by multiplying the answer by 41,253 to get 6,400 x 41,253 = 264,000,000 stars!

Problem 4 – Why do you think we needed to average the numbers in Problem 2? Answer: Because stars are not evenly spread across the sky, so you need to figure out the average number of stars.

How many stars are in the sky?

One of the very first things that astronomers studied was the number of stars in the sky. From this, they hoped to get a mathematical picture of the shape and extent of the entire Milky Way galaxy. This is perhaps why some cartoons of 'astronomers' often have them sitting at a telescope and tallying stars on a sheet of paper! Naked-eye counts usually number a few thousand, but with increasingly powerful telescopes, fainter stars can be seen and counted, too.

Over the decades, 'star count' sophisticated models have been created, and rendered into approximate mathematical functions that let us explore what we see in the sky. One such approximation, which gives the average number of stars in the sky, is shown below:

$$\mathrm{Log}_{10}N(m) = -0.0003\ m^3 + 0.0019\ m^2 + 0.484\ m - 3.82$$

This polynomial is valid over the range [+4.0, +25.0] and gives the Log_{10} of the total number of stars per square degree fainter than an apparent magnitude of m. For example, at an apparent magnitude of +6.0, which is the limit of vision for most people, the function predicts that $\mathrm{Log}_{10}N(6) = -0.912$ so that there are $10^{-0.912} = 0.12$ stars per square degree of the sky. Because the full sky area is 41,253 square degrees, there are about 5,077 stars brighter than, or equal to, this magnitude.

Problem 1 - A small telescope can detect stars as faint as magnitude +10. If the human eye-limit is +6 magnitudes, how many more stars can the telescope see than the human eye?

Problem 2 - The Hubble Space Telescope can see stars as faint as magnitude +25. About how many stars can the telescope see in an area of the sky the size of the full moon (1/4 square degree)?

Problem 3 - A photograph is taken of a faint star cluster that has an area of 1 square degree. If the astronomer counts 5,237 stars in this area of the sky with magnitudes in the range from +11 to +15, how many of these stars are actually related to the star cluster?

Problem 1 - Answer: From the example, there are 0.12 stars per square degree brighter than +6.0

$$Log_{10}N(+10) = -0.0003\ (10)^3 + 0.0019\ (10)^2 + 0.484\ (10) - 3.82$$
$$= -0.3 + 0.19 + 4.84 - 3.82$$
$$= +0.55$$

So there are $10^{.55}$ = 3.55 stars per square degree brighter than +10. Converting this to total stars across the sky (area = 41,253 square degrees) we get 5,077 stars brighter than +6 and 146,448 stars brighter than +10. The number of additional stars that the small telescope will see is then 146,448 - 5,077 = **141,371 stars.**

Problem 2 - Answer: $Log_{10}N(25) = -0.0003\ (25)^3 + 0.0019\ (25)^2 + 0.484\ (25) - 3.82$
$= +4.78$

So the number of stars per square degree is $10^{+4.78}$ = 60,256. For an area of the sky equal to 1/4 square degree we get (60,256) x (0.25) = **15,064 stars**.

Problem 3 - Answer: $Log_{10}N(15)$ counts up all of the stars in an area of 1 square degree with magnitudes of 0, +1, +2,…+15. $Log_{10}N(10)$ counts up all of the stars in an area of 1 square degree with magnitudes of 0, +1, +2,…+10. The difference between these two will be the number of stars with magnitudes of +11, +12, +13, +15, which is the number of stars in the sky per square degree, in the magnitude range of the star cluster. We then subtract this number from 5,237 to get the number of stars actually in the cluster.

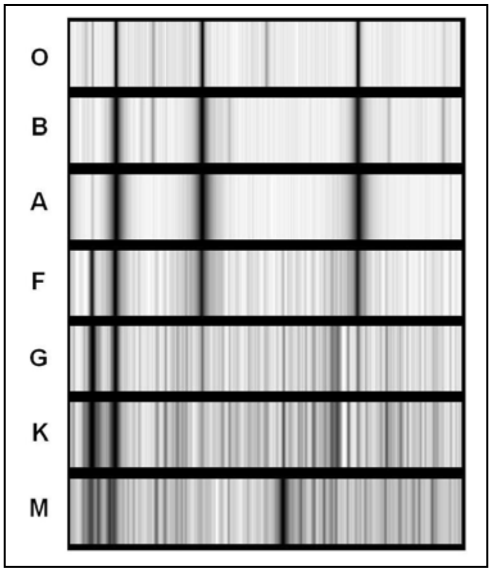
$Log_{10}N(15) = +2.86$ corresponds to $10^{+2.86}$ = 716.1 stars/square degree

$Log_{10}N(11) = +1.33$ corresponds to $10^{+1.33}$ = 21.6 stars/square degree

So the difference between these is 716.1-21.6 = 694.5 stars/deg^2. The star cluster area is 5 square degrees, so we have 694.5 stars/deg^2 x (5 square degrees) = 3,473 stars that we expect to find in the sky in the magnitude range of the star cluster. Since the astronomer counted 5,237 stars in the star cluster field, that means that 3,473 of these stars are probably just part of the general sky population of stars, while only 5,237 - 3,473 = **1,764 stars are actually members of the cluster** itself.

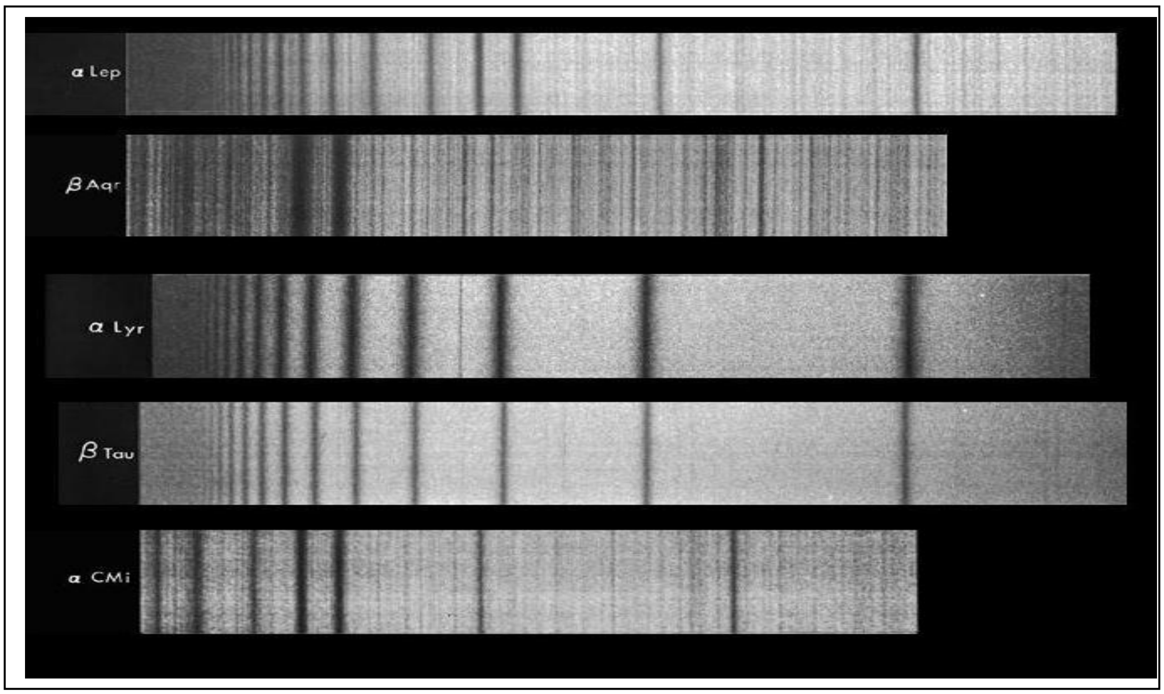
Note to Teacher: Show that the students have to evaluate N first before taking the difference because $Log_{10}N(15) - Log_{10}N(11)$ is not the same as N(15) - N(10).

Spectral Classification of Stars



The advent of the spectroscope in the 1800's allowed astronomers to study the temperatures and compositions of stars, and to classify stars according to their spectral similarities. At first, 26 classes were defined; one for each letter in the alphabet. But only 7 are actually major classes, and these survive today as the series 'O, B, A, F, G, K, M'. This series follows decreasing star temperatures from 30,000 K (O-type) to 3,000 K (M-type).

Images courtesy: Helmut Abt (NOAA).



Problem 1 - Sort the five stellar spectra according to their closest matches with the standard spectra at the top of the page. (Note, the spectra may not be to the same scale, aligned vertically, and may even be stretched!)

Problem 2 - The star α Lyr (Alpha Lyra) has a temperature of 10,000 K and β Aqr (Beta Aquarii) has a temperature of 5,000 K. What do you notice about the pattern of spectral lines as you change the star's temperature?

Problem 1 - This is designed to be a challenge! Student strategies should include looking for general similarities first. One obvious way to group the spectra in terms of the increasing (or decreasing!) number of spectral lines.

Alpha Lyr (Alpha Lyra) and Beta Tau (Beta Tauri) can be grouped together because of the strong lines that appear virtually alone in the spectra (these are hydrogen lines). The second grouping, Group 2, would include Alpha Lep (Alpha Leporis) and Alpha CMi (Alpha Canis Minoris) because they have a different pattern of strong lines than Group 1 but also the hint of many more faint lines in between. The last 'group' would be for Beta Aqr (Beta Aquarii) because it has two very strong lines close together (the two lines of the element calcium), but many more lines that fill up the spectrum and are stronger than in Group 2.

Group 1:

Vega (Alpha Lyra) - A type star
Alnath (Beta Tauri) - B type star

Group 2:

Arneb (Alpha Leporis) - F type star
Procyon (Alpha Canis Minoris) - F type star

Group 3:

Sadalsuud (Beta Aquarii) - G type star

Comparing Group 1, 2 and 3 with the standard chart, Group 1 consists of the B and A type stars, in particular Alpha Lyra has the thick lines of an A-type star, and Beta Tauri has the thinner lines of a B-type star; Group 2 is similar to the spectra of the F-type stars, and Group 3 is similar to the G-type stars.

Problem 2 - Vega has a temperature of 10,000 K and Beta Aquari has a temperature of 5,500 K. What do you notice about the pattern of spectral lines as you change the star's temperature?

Answer: What you should notice is that, as the temperature of the star gets cooler, the number of atomic lines in this part of the spectrum (visible light) increases.

The Most Massive Stars Known

Star	Mass (Sun=1)	Diameter (Sun=1)	Luminosity (Sun=1)
R136a1	350	35	10,000,000
Eta Carina	250	195	5,000,000
Peony Nebula	175	100	3,200,000
Pistol Star	150	340	6,300,000
HD269810	150	19	2,200,000
LBV1806	130	200	10,000,000
HD93129a	120	25	5,500,000
HD93250	118	18	5,000,000
S Doradus	100	380	600,000

The picture above shows the Tarantula Nebula in the Large Magellanic Cloud, located 165,000 light years from Earth. Astronomers using data from the Hubble Space Telescope and the European Space Observatory's Very Large Telescope in Chile have recently determined that it harbors the most massive known star in our 'corner' of the universe which they call R136a1. The table lists some of the most massive known stars as of July, 2010.

Problem 1 - From the indicated sizes relative to the Sun, create a scale model that shows the relative sizes of these stars compared to the Sun, assuming that in the scaled model the solar disk has a diameter of 1 millimeter.

Problem 2 - The predicted lifespan of a star depends on its luminosity according to the formula

$$T = \frac{10 \text{ billion years}}{M^{2.5}}$$

For the Sun, M = 1 and so its lifespan is about 10 billion years. A star with 10 times the mass of our Sun, M=10, will last about 30 million years! From the table above, what are the predicted life spans for these 'hypergiant' stars to two significant figures?

Problem 3 - How many generations of a 100 solar-mass hypergiant could pass during the life span of a single star like our own Sun?

Problem 1 - From the indicated sizes relative to the Sun, create a scale model that shows the relative sizes of these stars compared to the Sun, assuming that in the scaled model the solar disk has a diameter of 1 millimeter.

Answer: **Example of calculation: R136a1 diameter = 700x sun, so 700 x 1 mm = 0.7 meters in diameter!**

Problem 2 - The predicted lifespan of a star depends on its luminosity according to the formula

$$T = \frac{10 \text{ billion years}}{M^{2.5}}$$

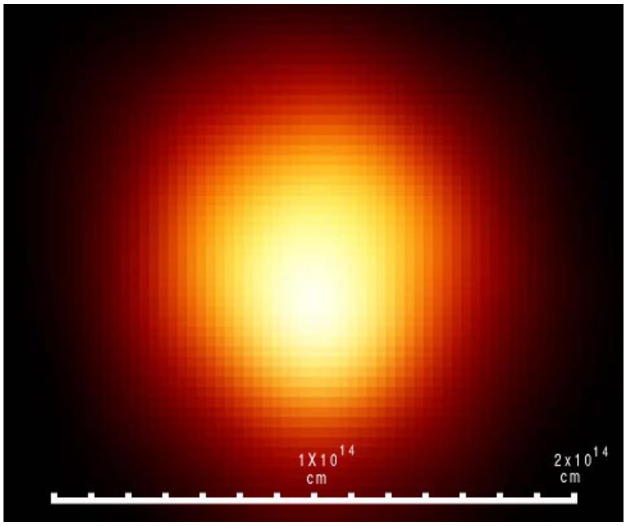
For the Sun, M = 1 and so its lifespan is about 10 billion years. A star with 10 times the mass of our Sun, M=10, will last about 30 million years! From the table above, what are the predicted lifespans for these 'hypergiant' stars?

Answer:

Star	Mass (Sun=1)	Diameter (Sun=1)	Luminosity (Sun=1)	Lifespan (years)
R136a1	350	35	10,000,000	**4,400**
Eta Carina	250	195	5,000,000	**10,000**
Peony Nebula	175	100	3,200,000	**25,000**
Pistol Star	150	340	6,300,000	**36,000**
HD269810	150	19	2,200,000	**36,000**
LBV1806	130	200	10,000,000	**52,000**
HD93129a	120	25	5,500,000	**63,000**
HD93250	118	18	5,000,000	**66,000**
S Doradus	100	380	600,000	**100,000**

Problem 3 - How many generations of a 100 solar-mass hypergiant could pass during the life span of a single star like our own Sun?

Answer: Life span of our sun is 10 billion years. Life span of a 100 solar-mass hypergiant is 100,000 years, so about 10 billion/100,000 = **100,000 generations** could come and go.



The amount of power that a star produces in light is related to the temperature of its surface and the area of the star. The hotter a surface is, the more light it produces. The bigger a star is, the more surface it has. When these relationships are combined, two stars at the same temperature can be vastly different in brightness because of their sizes.

Image: Betelgeuse (Hubble Space Telescope.) It is 950 times bigger than the sun!

The basic formula that relates stellar light output (called luminosity) with the surface area of a star, and the temperature of the star, is L = A x F where the star is assumed to be spherical with a surface area of $A = 4 \pi R^2$, and the radiation emitted by a unit area of its surface (called the flux) is given by $F = \sigma T^4$. The constant, σ, is the Stefan-Boltzman radiation constant and it has a value of $\sigma = 5.67 \times 10^{-5}$ ergs/ (cm^2 sec deg^4). The luminosity, L, will be expressed in power units of ergs/sec if the radius, R, is expressed in centimeters, and the temperature, T, is expressed in degrees Kelvin. The formula then becomes,

$$L = 4 \pi R^2 \sigma T^4$$

Problem 1 - The Sun has a temperature of 5700 Kelvins and a radius of 6.96×10^5 kilometers, what is its luminosity in A) ergs/sec? B) Watts? (Note: 1 watt = 10^7 ergs/sec).

Problem 2 - The red supergiant Antares in the constellation Scorpius, has a temperature of 3,500 K and a radius of 700 times the radius of the sun. What is its luminosity in A) ergs/sec? B) multiples of the solar luminosity?

Problem 3 - The nearby star, Sirius, has a temperature of 9,200 K and a radius of 1.76 times our Sun, while its white dwarf companion has a temperature of 27,400 K and a radius of 4,900 kilometers. What are the luminosities of Sirius-A and Sirius-B compared to our Sun?

Calculus*:*

Problem 4 - Compute the total derivative of L(R,T). If a star's radius increases by 10% and its temperature increases by 5%, by how much will the luminosity of the star change if its original state is similar to that of the star Antares? From your answer, can you explain how a star's temperature could change without altering the luminosity of the star. Give an example of this relationship using the star Antares!

Answer Key

Problem 1 - We use $L = 4\ (3.141)\ R^2\ (5.67 \times 10^{-5})\ T^4$ to get L (ergs/sec) = 0.00071 R(cm)^2 T(degreesK)^4 then,

A) L(ergs/sec) = 0.00071 x (696,000 km x 10^5 cm/km)^2 (5700)^4 = **3.6 x 10^{33} ergs/sec**

B) L(watts) = 3.6 x 10^{33} (ergs/sec)/ 10^7 (ergs/watt) = **3.6 x 10^{25} watts**.

Problem 2 - A) The radius of Antares is 700 x 696,000 km = 4.9 x 10^8 km.
L(ergs/sec) = 0.00071 x (4.9 x 10^8 km x 10^5 cm/km)^2 (3500)^4 = **2.5 x 10^{38} ergs/sec**
B) L(Antares) = (2.5 x 10^{38} ergs/sec) / (3.6 x 10^{33} ergs/sec) = **71,000 L(sun).**

Problem 3 - Sirius-A radius = 1.76 x 696,000 km = 1.2 x 10^6 km
L(Sirius-A) = 0.00071 x (1.2 x 10^6 km x 10^5 cm/km)^2 (9200)^4 = **7.3 x 10^{34} ergs/sec**
L = (7.3 x 10^{34} ergs/sec) / (3.6 x 10^{33} ergs/sec) = **20.3 L(sun).**
L(Sirius-B) = 0.00071 x (4900 km x 10^5 cm/km)^2 (27,400)^4 = **9.5 x 10^{31} ergs/sec**
L(Sirius-B) = 9.5 x 10^{31} ergs/sec / 3.6 x 10^{33} ergs/sec = **0.026 L(sun).**

Advanced Math:

Problem 4 *(*Note: In the discussion below, the symbol d represents a partial derivative)

$$dL(R,T) = \frac{dL(R,T)}{dR}\, dR + \frac{dL(R,T)}{dT}\, dT$$

$$dL = [\,4\pi\ (2)\ R\sigma T^4\,]\ dR + [\,4\pi\ (4)\ R2\,\sigma T^3\,]\ dT$$

$$dL = 8\pi\ R\sigma T^4\ dR + 16\pi\ R^2\sigma T^3\ dT$$

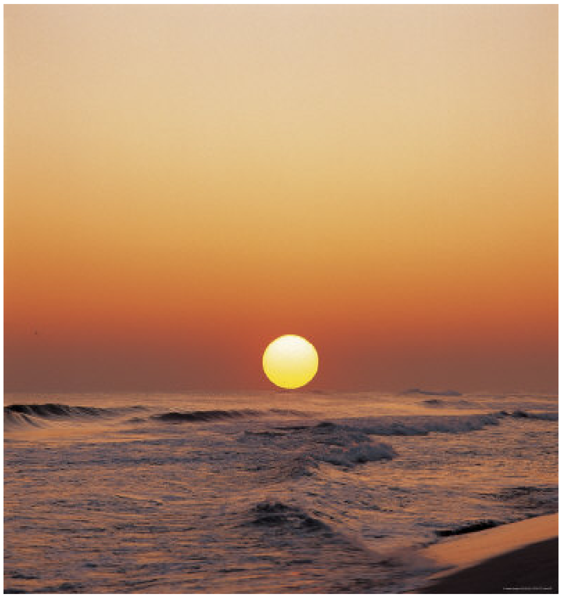
To get percentage changes, divide both sides by $L = 4\pi\ R^2\sigma T^4$

$$\frac{dL}{L} = \frac{8\pi\ R\sigma T^4}{4\pi\ R^2\sigma T^4}\, dR + \frac{16\pi\ R^2\sigma T^3}{4\pi\ R^2\sigma T^4}\, dT$$

Then dL/L = 2 dR/R + 4 dT/T so for the values given, dL/L = 2 (0.10) + 4 (0.05) = **0.40 The star's luminosity will increase by 40%.**

Since dL/L = 2 dR/R + 4 dT/T, we can obtain no change in L if 2 dR/R + 4 dT/T = 0. This means that 2 dR/R = - 4 dT/T and so, - 0.5 dR/R = dT/T. **The luminosity of a star will remain constant if, as the temperature decreases, its radius increases.**

Example. For Antares, its original luminosity is 71,000 L(sun) or 2.5 x 10^{38} ergs/sec. If I increase its radius by 10% from 4.9 x 10^8 km to 5.4 x 10^8 km, its luminosity will remain the same if its temperature is decreased by dT/T = 0.5 x 0.10 = 0.05 which will be 3500 x 0.95 = 3,325 K so L(ergs/sec) = 0.00071 x (5.4 x 10^8 km x 10^5 cm/km)^2 (3325)^4 = **2.5 x 10^{38} ergs/sec**



The Milky Way galaxy has an estimated 500 billion stars, but most of them are quite unlike our own, warm, yellow sun.

There are red dwarfs that shine with only 1/1000 our Sun's intensity.

There are massive stars that shine with over one million time our sun's intensity.

We know that life has emerged on a planet orbiting our own warm, yellow star, so it is natural for us too look for these stars first when searching for possible life-baring worlds.

Astronomers classify stars on a temperature scale with the letters, O, B, A, F, G, K and M. Our own sun is a G-type dwarf star with a temperature of 6,000 K. The O, B stars are luminous with temperatures over 15,000 K. The A-type stars, such as Vega, are 10,000 K. Then we have M-type stars that shine a crimson red at 3,000 K. The stars with classes of F, G and K span temperatures from 9,000 to 4,000 K and are broadly considered sun-like.

The attached page is from the *Hipparcos Input Catalog* developed by the European Space Agency. Although the table lists only 95 stars, the full catalog is a list of the 118,209 brightest stars in the sky, which were studied by the Hipparcos satellite to determine their exact locations and distances from our Sun. In preparation for the satellite study, this basic catalog was developed to provide information such as the star's name, position in the sky, motion, and brightness. The catalog also included the 'Spectral Type' of the star in column 5, labeled *Sp. Type*.

Problem 1 – The spectral classes are indicated by the letters O, B, A, F, G, K and M. Create a frequency table that gives the number of stars in the table that have been classified in each of the 7 categories. Also include an eighth category 'Unknown' for those stars that have no classification.

Problem 2 – Our sun is a G-type star. What percentage of the stars in this table are G-type?

Problem 3 – How many G-type stars might you expect to find in the full Hipparcos Input Catalog?

Problem 4 – There are 500 billion stars in our Milky Way. If the Hipparcos Input Catalog represents a fair, random, sample of the Milky Way, how many G-type stars might you expect to find in our Milky Way galaxy?

Problem 1 – The spectral classes are indicated by the letters O, B, A, F, G, K and M. Create a frequency table that gives the number of stars in the table that have been classified in each of the 7 categories. Also include an eighth category 'Unknown' for those stars that have no classification.

Answer:

Type	Number
O	**0**
B	**8**
A	**18**
F	**20**
G	**18**
K	**20**
M	**7**
Unknown	**4**

Problem 2 – Our sun is a G-type star. What percentage of the stars in this table are G-type?

Answer: 100% x (18/95) = **19%**

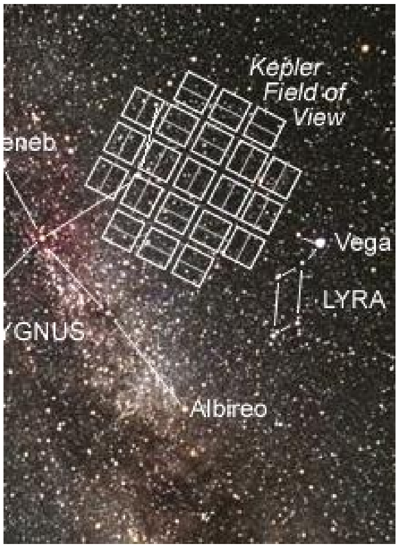
Problem 3 – How many G-type stars might you expect to find in the full Hipparcos Input Catalog?

Answer: 118,209 x 0.19 = **22,459**.

Note, if the percentage is rounded to 2 significant figures ('19') then the answer should be quoted to 2 significant figures as well.. **22,000**. This could be suggested as an 'extra credit' response.

Problem 4 – There are 500 billion stars in our Milky Way. If the Hipparcos Input Catalog represents a fair, random, sample of the Milky Way, how many G-type stars might you expect to find in our Milky Way galaxy?

Answer: 500 billion x 0.19 = **95 billion G-type stars**.



The Kepler Survey Field grid is located between the constellations Cygnus and Lyra. Although millions of stars are in this area, only 157,000 are studied by Kepler.

The sky is filled with millions of stars, but in order to carry out a survey to search for interesting planets, you want to select the stars you will be studying by using a set of statistically consistent measures of their interesting properties.

The goal is that, after you have exhaustively studied your small sample, you can make a reasonably good estimate of how many are yet to be found in the much larger sample of stars….by some estimations, over 500 billion in the Milky Way!

The attached table is taken from a list of 1000 stars in the Kepler Survey Catalog located within 10 arcminutes of the sky coordinates RA(2000) = 19h 20m 00s and Dec(2000) = +51^{o} 20' 00". The 1000 stars were pre-selected by the Kepler Science Team because they are brighter than +20 in the spectral region studied by the Kepler instruments.

The Kepler Data Table consists of 254 stars, and is much shorter than the 1000 stars in the original list because these are the stars for which the temperatures have been determined so far. The table below classifies stars according to their temperatures. A G-type star like our sun has a surface temperature between 5,200 K to 6,000 K.

Spectral Class	Temperature Range (K)
O	> 33,000
B	10,000 to 32,999
A	7,500 to 9,000
F	6,000 to 7,499
G	5,200 to 5,999
K	3,700 to 5,199
M	< 3,699

Problem 1 – From the Kepler Data Table, classify each star based on its cataloged temperature in columns 3, 6 and 9.

Problem 2 – How many of the 254 stars are found in each of the seven spectral types?

Problem 3 – To the nearest hundred, about how many stars in the full Kepler Survey of 157,000 stars will be a sun-like G-type star?

(1)	(2)	(3)		(4)	(5)	(6)		(7)	(8)	(9)
ID	Mag.	Temp.		ID	Mag.	Temp.		ID	Mag.	Temp.
12457780	9	4300		12507925	16	6350		12457691	16	5542
12405425	10	6086		12457928	16	6219		12405343	16	5539
12508094	11	6666		12405321	16	3809		12405283	16	5038
12458003	12	4601		12457962	16	5091		12405202	16	5985
12457926	12	5034		12405215	16	5722		12457591	16	6014
12507830	12	5885		12457636	16	4727		12458060	17	4931
12457978	12	6711		12457990	16	6131		12457726	17	5344
12458116	13	4609		12457847	16	6173		12457882	17	4971
12404954	13	5706		12457717	16	4841		12508216	17	5603
12457754	13	6242		12457646	16	6485		12457714	17	5669
12405158	13	4934		12405275	16	4897		12405259	17	5527
12458073	14	5059		12457936	16	5532		12457734	17	5186
12457812	14	5210		12405167	16	5943		12457787	17	5934
12405226	14	4983		12457959	16	5868		12458058	17	5886
12404984	14	4943		12457931	16	6416		12405089	17	5116
12457684	14	5133		12404902	16	5358		12405227	17	5805
12457741	14	6353		12457918	16	5742		12457884	17	4937
12405195	14	6008		12508192	16	5931		12457695	17	5891
12457989	14	5948		12405182	16	5205		12458096	17	6163
12508103	14	5848		12405231	16	5784		12458121	17	5411
12457730	14	5935		12457667	16	6041		12508111	17	5216
12457993	14	5923		12457666	16	5407		12457763	17	5155
12458078	14	5826		12405159	16	4834		12457608	17	5941
12458054	14	6085		12405133	16	5786		12457992	17	5890
12508067	15	6219		12457587	16	5956		12458030	17	5927
12457878	15	6111		12405342	16	6295		12405330	17	6215
12457664	15	5685		12457719	16	6234		12405015	17	4312
12405172	15	6281		12457565	16	6148		12405310	17	5332
12457628	15	5619		12457746	16	6007		12404892	17	5739
12508078	15	5804		12405234	16	5884		12457861	17	6378
12458105	15	5825		12457571	16	5888		12458039	17	6113
12405197	15	5424		12457835	16	11143		12457982	17	5969
12458099	15	5982		12508155	16	5487		12508205	17	4891
12457786	15	4813		12405160	16	4756		12457765	17	5080
12405364	15	5972		12457897	16	5960		12457792	17	6289
12507871	15	5007		12457696	16	4846		12405322	17	4337
12405211	15	6430		12457839	16	6448		12457699	17	5949
12405139	15	6160		12458029	16	3963		12405324	17	6141
12405132	15	4869		12457789	16	5401		12457663	17	6087
12457798	15	5009		12507981	16	4981		12508028	17	6024
12457968	15	5624		12457755	16	4865		12457733	17	4101
12507905	15	5184		12457920	16	4844		12404932	17	5851
12405192	15	5226		12457616	16	5972		12405162	17	5096
12507890	15	4780		12457642	16	5713		12458070	17	5191
12457762	15	4778		12457724	16	3799		12404924	17	6158
12405306	15	5662		12405055	16	5975		12457771	17	5525
12457709	15	5405		12457577	16	4376		12508041	17	6010
12404934	15	4987		12457869	16	5389		12457648	17	3870
12405285	15	5413		12457930	16	5690		12457704	17	5341
12404979	15	6204		12405193	16	5881		12405350	17	4957

(1)	(2)	(3)		(4)	(5)	(6)		(7)	(8)	(9)
ID	Mag.	Temp.		ID	Mag.	Temp.		ID	Mag.	Temp.
12457973	17	5072		12508248	18	5684		12405019	18	4671
12457542	17	5878		12457853	18	5687		12405356	18	5858
12508139	17	5834		12508050	18	5778		12457692	18	5708
12458000	17	4866		12457971	18	5819		12405176	18	5033
12405334	17	4228		12458081	18	6225		12457621	18	5327
12508043	17	5671		12457672	18	5184		12405069	18	4372
12457725	17	6240		12457543	18	6234		12405268	18	4579
12404993	17	3550		12457813	18	6004		12457922	18	4517
12405236	17	5706		12457960	18	5810		12405138	18	4912
12457595	17	5596		12457919	18	6164		12457657	18	5727
12457872	17	5440		12457645	18	5893		12457986	18	4588
12405237	17	5882		12405087	18	5783		12457652	18	5017
12508065	17	6000		12405267	18	6194		12405392	18	5786
12457606	17	5845		12508048	18	6157		12458062	18	5818
12405362	17	5305		12405150	18	5738		12458074	18	5854
12457596	17	6665		12457610	18	5806		12508106	18	4824
12404965	17	5803		12457924	18	5717		12405219	18	5113
12457690	17	5292		12457844	18	6723		12457593	18	6449
12457632	17	6263		12457975	18	5169		12457795	18	6138
12457702	17	5347		12458065	18	5084		12457943	18	5081
12405435	17	5872		12507896	18	6061		12405235	18	5526
12457977	17	6420		12457834	18	4708		12458091	18	5709
12405257	17	5566		12458083	18	5645		12405339	18	4756
12457578	17	5080		12405080	18	5197		12405326	18	5763
12457569	17	5247		12457745	18	5134		12507947	18	6020
12457862	17	6612		12405110	18	5734		12508115	18	6387
12405346	17	6068		12458055	18	5801				
12457880	17	4427		12457938	18	5580				
12405305	17	5638		12405223	18	5971				
12405278	17	4569		12457958	18	6354				
12508151	17	5394		12457963	18	5228				
12508173	17	4615		12457868	18	5905				
12404982	17	5042		12457681	18	5281				
12508052	17	5508		12457740	18	5465				
12457732	17	4185		12405092	18	5462				
12457987	17	5648		12405218	18	4270				
12405074	17	5547		12404944	18	6019				
12458082	17	4451		12457738	18	5549				
12508186	17	5908		12458127	18	5915				

The Kepler Table was created by visiting the Kepler Target Search Portal at http://archive.stsci.edu/kepler/kepler_fov/search.php
The search window menu has five entry boxes at the top of the form. Leave all the windows blank except 'Right Ascension' for which you enter 19h 20m 00s and in the Declination box enter +51d 20' 00". In the Radius (arcmin) box enter 10 These entries will have the interface pull out all of the Kepler Target Stars within 10 arcminutes of the sky coordinate you entered in the Right Ascension and Declination boxes. At the bottom of the form where an input box says 'Output Columns', use the right-hand buttons adjacent to the box to delete all of the possible column parameters except 'Kepler_ID', 'KEP_Mag' and 'Teff(degK)'. Then in the 'Output Format' window select the Excel_spreadsheet' option. Click on the 'Search' button at the top of the page and the program will create an Excel spreadsheet whose first three columns are Kepler_ID, KEP_Mag and Teff(DegK). This table has 1000 stars, however, only the 254 stars in the attached table have known temperatures at this time, so this is the sample we are working with in these problems.

Problem 1 – From the Kepler Data Table, classify each star based on its cataloged temperature in columns 3, 6 and 9.

Example: ID = 12457973 T = 5072 which is between 3,700 and 5,199 so it is K-type.

Problem 2 – How many of the 254 stars are found in each of the seven spectral types?

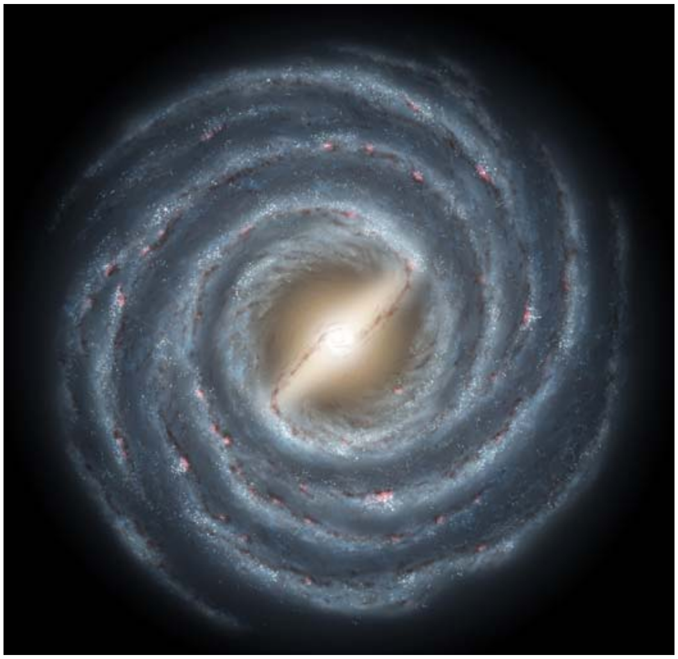
Answer:

Spectral Class	Temperature Range (K)	Number
O	> 33,000	**0**
B	10,000 to 32,999	**1**
A	7,500 to 9,000	**0**
F	6,000 to 7,499	**57**
G	5,200 to 5,999	**124**
K	3,700 to 5,199	**71**
M	< 3,699	**1**

Problem 3 – To the nearest hundred, about how many stars in the full Kepler Survey of 157,000 stars will be a sun-like G-type star?

Answer: Out of a total of 254 Kepler-selected stars, 124 were G-type or F = 124/254 = 0.488, then assuming that this sample of 254 cataloged stars is representative of the entire catalog of 157,000 stars, there should be about N = 157,000 x 0.488 = 76,616 G-type stars. To the nearest hundred, this is **76,600 stars**.
Note, the Kepler Survey stars are specifically selected to be in one of three stellar classes F, G or K.

Estimating the number of stars in the Milky Way



Although we can estimate the number of stars we see in the sky from our vantage point on Earth, it is much harder to estimate the total number in such a vast galaxy as ours. The Milky Way is a flattened disk-like collection of stars and matter over 100,000 light years in diameter and 100 light years thick.

One thing we can do is determine the total mass of our galaxy, and then divide this by the average mass of a single star. Yet even this method is not without its uncertainties.

Problem 1 - From Kepler's Third Law, the mass of an object, M, is related to the orbital period and distance of its satellite according to the Newtonian formula:

$$M = \frac{4\pi^2 R^3}{GT^2}$$

A) If Earth takes 31 million seconds to travel once around the sun in its orbit, and the orbit has a radius of 150 billion meters, what is the mass of our sun if the constant of gravity, G = 6.67×10^{-11} ? B) If our sun takes 250 million years to travel once around the center of the Milky Way, and is located 26,000 light years from the Galactic Center, about what is the estimated mass of all of the matter inside the orbit of the sun? (Note 1 light year = 9.5×10^{15} meters).

Problem 2 - A survey of 30 nearby stars gives the following table, where the mass is given in multiples of our sun's mass. What is the average mass of a star in the sun's neighborhood?

Mass	Mass	Mass	Mass	Mass	Mass	Mass	Mass
0.85	0.04	0.14	1.77	1.1	0.31	0.4	0.27
1.00	0.8	0.08	0.63	0.9	0.25	0.4	0.16
0.52	0.43	2.31	0.35	0.38	0.92	0.63	
0.04	0.21	0.98	0.1	0.38	0.69	0.6	

Problem 3 - Using a sophisticated version of the above method, astronomers have established that the total mass of the Milky Way is about 1 trillion times the mass of our sun. Of this, only 40% is in the form of actual stars. From your answer to Problem 2, about how many stars are in the Milky Way?

Illustration Credit: R. Hurt (SSC), JPL-Caltech, NASA

The most recent estimate from a study using information from the Sloan Digital Sky Survey measuring the velocity of over 2,400 stars put the mass of the Milky Way and its halo at 1 trillion solar masses. (http://www.sdss.org/news/releases/20080527.mwmass.html) within 190,000 light years of its center.

Problem 1 - From Kepler's Third Law, the mass of an object, M, is related to the orbital period and distance of its satellite according to the Newtonian formula:

$$M = \frac{4\pi^2 R^3}{GT^2}$$

A) If Earth takes 31 million seconds to travel once around the sun in its orbit, and the orbit has a radius of 150 billion meters, what is the mass of our sun if the constant of gravity, G = 6.67 x 10^{-11} ?

Answer: $M = \frac{4(3.141)^2 \left(1.5x10^{11}\right)^3}{(6.67x10^{-11})(3.1x10^{7})^2}$ = **2.1 x 10^{30} kilograms**

B) If our sun takes 250 million years to travel once around the center of the Milky Way, and is located 26,000 light years from the Galactic Center, about what is the estimated mass of all of the matter inside the orbit of the sun? (Note 1 light year = 9.5 x 10^{15} meters).
Answer; R = 26000 (9.5 x 10^{15}) = 2.5 x 10^{20} meters.
T = 2.5x10^8 years x (3.1x10^7 seconds/year) = 7.8x10^{15} sec

$M = \frac{4(3.141)^2 \left(2.5x10^{20}\right)^3}{(6.67x10^{-11})(7.8x10^{15})^2}$ = **1.5 x 10^{41} kg or 76 billion solar masses**.

Problem 2 - A survey of 30 nearby stars gives the following table, where the mass is given in multiples of our sun's mass. What is the average mass of a star in the sun's neighborhood?

Mass	Mass	Mass	Mass	Mass	Mass	Mass	Mass
0.85	0.04	0.14	1.77	1.1	0.31	0.4	0.27
1.00	0.8	0.08	0.63	0.9	0.25	0.4	0.16
0.52	0.43	2.31	0.35	0.38	0.92	0.63	
0.04	0.21	0.98	0.1	0.38	0.69	0.6	

Answer: The sum of the 30 masses is 17.64 so the average is just M = 17.64/30 = **0.59 solar masses.**

Problem 3 - Using a sophisticated version of the above method, astronomers have established that the total mass of the Milky Way is about 1 trillion times the mass of our sun. Of this, only 40% is in the form of actual stars. From your answer to Problem 2, about how many stars are in the Milky Way? Answer: The estimated stellar mass of the Milky Way is 0.4 x 1 trillion = 400 billion solar masses. If we divide this by the average mass of a star using our solar neighborhood as a typical sample, we get

N = 400 billion solar masses x (1 star / 0.59 solar masses)
= **678 billion stars.**

$$N = N^* f_p N_e f_l f_i f_c \left(\frac{L}{T_g} \right)$$

N^* = number of stars in Milky Way
f_p = fraction of stars with planets
n_e = average number of habitable planets per star
T_g = age of Milky Way
f_l = fraction of habitable planets that have life
f_i = fraction that develop intelligent life
f_c = fraction of civilizations that develop interstellar communication
L = lifetime of detectable intelligent emission

The Drake Equation is a simple formula whose factors, when multipled together, provide an estimate for the number of civilizations in the Milky Way that exist at the present time, and are able to communicate across interstellar distances.

The first four factors N^*, f_p, n_e and T_g, are astronomical in nature, and require knowledge about our Milky Way, and the expected number of planets located in their Habitable Zones. These factors can in principle be determined through current astronomical observation programs.

The next four factors depend on our detailed understanding of the origin of life and its evolution towards intelligence. At the present time, the proper values to use are maters of intense speculation, with no apparent 'correct' answers.

The astronomical factors have, largely, been determined, and their values can be deduced from your answers to previous problems in this book. The age of the Milky Way is generally considered to be about 12.5 billion years based on detailed studies of the ages of its various stellar populations.

Problem 1 - N^* : What is the best current estimate for the number of stars in the Milky Way?

Problem 2 - f_p, : NASA's Kepler mission has recently surveyed a large number of stars and detected hundreds of planet candidates. What is our current best estimate for the fraction of stars that possess planets?

Problem 3 - n_e : A planet can be considered habitable if it is located in an orbit far enough from its star that liquid water could be present. It also requires that the planet have an atmosphere, though not necessarily one that is breathable by humans. Many earth bacteria find oxygen a lethal poison. From the Kepler survey, about how many planets might typically exist it their star's habitable zone?

Problem 4 - Combine the values for the four astronomical factors according to the formula above, and determine the value for Da - the astronomical factor for the Drake Equation.

Problem 1 - N* : What is the best current estimate for the number of stars in the Milky Way?

Answer: According to your answer for '*Estimating the Number of Stars in the Milky Way*' the value is **N* = 678 billion stars**.

Problem 2 - NASA's Kepler mission has recently surveyed a large number of stars and detected hundreds of planet candidates. What is our current best estimate for the fraction of stars that possess planets?

Answer: According to the answers for '*Earth-like Planets by the Score*' ,Kepler surveyed 156,453 stars and has so-far detected 1,235 planet candidates, of which 68 were earth-like in size, and 5 were located within their habitable zones. The fraction of the surveyed stars with planets, f_p, is therefore 1235/156453 or $\mathbf{f_p = 0.0079}$.

Note: Recent, systematic, ground-based surveys have studied 1,800 stars for over 10 years and detected 90 planets, so in this case fp = 90/1800 = 0.05, however other exoplanet surveys ,such as the HARPS Program, suggest that the faction may be closer to 0.30 when small, and hard to detect, earth-sized planets are included.

Problem 3 - n_e : A planet can be considered habitable if it is located in an orbit far enough from its star that liquid water could be present. It also requires that the planet have an atmosphere, though not necessarily one that is breathable by humans. Many earth bacteria find oxygen a lethal poison. From the Kepler survey, about how many planets might typically exist it their star's habitable zone?

Answer: According to your answer for '*Estimating the Number of Stars in the Milky Way*' the average number of the detected planets that are in their habitable zones, **Ne, is about 1**.

Note: We have eliminated giant planets in the habitable zones of their stars that might have orbiting satellites big enough to be planets (e.g Jupiters satellites Europa, Ganymede and Callisto are nearly planet-sized). The Kepler survey found 5 earth-sized planets in the habitable zones of their stars, but no cases, yet, where more than one of these earth-sized planets were found. Kepler found one planetary system Kepler-11, with as many as six planets of which 3 were in the habitable zone, so although N_e = 1 is consistent with current Kepler data, there may be a few cases for which N_e = 2 or 3. The average value for N_e however is probably close to 1.

Problem 4 - Combine the values for the four astronomical factors according to the formula above, and determine the value for Da - the astronomical factor for the Drake Equation.

Answer: $D_A = N^* f_p N_e \left(\frac{1}{T_g}\right)$ $\qquad D_A = (6.78x10^{11})(0.0079)(1.0)\left(\frac{1}{1.25x10^9}\right)$

So $\mathbf{D_A}$ **= 4.3 habitable planets / year**.

$$N = D_A f_l f_i f_c L$$

D_A = Astronomical Drake Factor

f_l = fraction of habitable planets that have life

f_i = fraction that develop intelligent life

f_c = fraction of civilizations that develop interstellar communication
L = lifetime of detectable intelligent emission
Tg = age of Milky Way

The Drake Equation is a simple formula whose factors, when multipled together, provide an estimate for the number of civilizations in the Milky Way that exist at the present time, and are able to communicate across interstellar distances.

The first factors D_A, depends on astronomical factors that have, largely, been determined, and their values can be deduced from your answers to previous problems in this book. From the previous exercise '*The Drake Equation: Astronomical Factors*' we deduced a value of about 4.3 habitable planets/year using data from ground-based and NASA Kepler planet surveys.

The next four 'biological' factors depend on our detailed understanding of the origin of life and its evolution towards intelligence. At the present time, the proper values to use are maters of intense speculation, with no apparent 'correct' answers. We have only our own Earth, and the history of life on this planet, to serve as an 'average' guide to life elsewhere.

Problem 1 - f_l : For our solar system, we have three planets that are in or near the habitable zone. What is your estimate for this factor assuming our solar system is typical?

Problem 2 - f_i : For our solar system, we have one planet in which life emerged. What is your estimate for this factor assuming our solar system is typical?

Problem 3 - f_c : For our solar system, we have one planet with intelligent life, and that life went on to develop radio technology. What is your estimate for this factor assuming our solar system is typical?

Problem 4 - L : For our solar system humans developed radio technology. How many years, L, do you think we will survive to continue to use radio technlogy that can be detected at interstellar distances?

Problem 5 - From the Drake equation above, with D_A = 4.3, and to two significant figures, what is your estimate for N : the number of intelligent civilizations in the Milky Way today that we might be able to detect or communicate?

Problem 1 - f_l : For our solar system, we have three planets that are in or near the habitable zone. What is your estimate for this factor assuming our solar system is typical? Answer: $\mathbf{f_l = 1/3}$

Problem 2 - f_i : For our solar system, we have one planet in which life emerged. What is your estimate for this factor assuming our solar system is typical? Answer: $\mathbf{f_i = 1.0}$

Problem 3 - f_c : For our solar system, we have one planet with intelligent life, and that life went on to develop radio technology. What is your estimate for this factor assuming our solar system is typical? Answer: $\mathbf{f_c = 1.0}$

Problem 4 - L : For our solar system humans developed radio technology. How many years, L, do you think we will survive to continue to use radio technology that can be detected at interstellar distances? Answer: **L = 100 years** includes current history of radio technology from 1910 to 2010. Students may debate whether we will continue to broadcast radio waves into space as we do today, as more of our data is transmitted by fiber optic cables. Other issues can also be discussed such as the catastrophic end of civilization from asteroid or comet impacts or other man-made calamities.

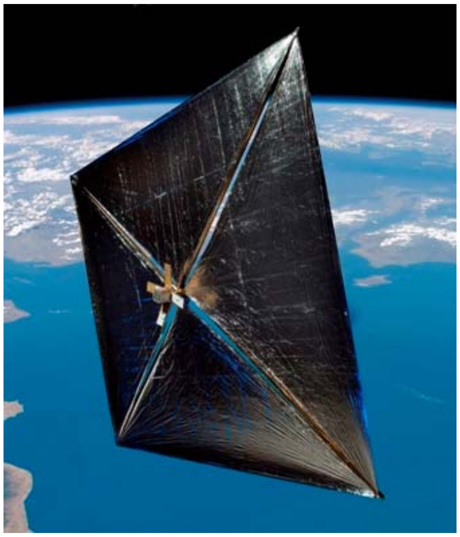
Problem 5 - From the Drake equation above, with D_A = 4.3, and to two significant figures, what is your estimate for N : the number of intelligent civilizations in the Milky Way today with which we might be able to detect or communicate?

Answer:

$$N = D_A f_l f_i f_c L$$

$$N = (4.3)(1/3)(1)(1)(100) = 143$$

so to 2 significant figures **N = 140 civilizations**


Solar Sail technology: Speed = 5000 k/s

Science fiction stories are based on the premise that humans will eventually figure out how to travel rapidly between the stars. At the present time, however, the only technologies we have tested suggest that these trips may be inconceivably long. As we detect more exoplanets, we are sure to discover some close-by that may harbor signs of life, and so increase the interest in sending an unmanned or even a manned mission to study extra-terrestrial life. How long would such journeys take, and what might be the problems we face in carrying them out?

Table of a few of the nearest known planets beyond our solar system

Name	Constellation	Distance (light years)	Distance from its star (AUs)	Mass (Earth=1)	Orbit Period
Epsilon Eridani b	Eridanus	10.5	5.8	475	6.9 years
Gleise 581g	Libra	20.3	0.14	3.1	36 days
Gleise 674	Ara	14.8	0.04	12	4.7 days
Gleise 876 d	Aquarius	15	0.02	8	1.9 days
Gleise 832 b	Grus	16.1	3.4	150	9.3 years
Gleise 176	Taurus	31	0.07	25	8.7 days
Fomalhaut b	Pices Austr.	25	115	600	872 years
61 Virginis b	Virgo	28	0.05	5.1	4.2 days

Problem 1 - The fastest spacecraft that have been launched to date are Unmanned: New Horizons (58,320 km/hr) and Manned: Apollo 10 (39,896 km/hr). What are the fastest travel times, in years, for an unmanned and a manned trip to a nearby extrasolar planet, if 1 light year = 9.3 trillion kilometers?

Problem 2 - Engineers have speculated that it might be possible to build a 'Ram Augmented Interstellar Rocket' that can reach a speed of 1,000 km/sec or greater. Using this technology, what is the maximum spacecraft travel time to each of the stars in the exoplanet list?

Problem 1 - The fastest spacecraft that have been launched to date are Unmanned: New Horizons (58,320 km/hr) and Manned: Apollo 10 (39,896 km/hr). What are the fastest travel times, in years, for an unmanned and a manned trip to a nearby extrasolar planet, if 1 light year = 9.3 trillion kilometers?

Answer: The nearest extrasolar planet is Epsilon Eridani b at a distance of 10.5 light years. This is equal to 10.5 light years x (9.3 x 10^{12} km / 1 light year) = 9.8×10^{13} kilometers.

Unmanned; At a speed of 58,320 km/hr,
t = 9.8×10^{13} kilometers / 58,320 km/hr
t = 1.7×10^{9} hours.
T = 1.7×10^{9} hours x (1 day / 24 hours) x (1 year / 365 days) = **194,000 years**.

Manned: At a speed of 39,896 km/hr,
t = 9.8×10^{13} kilometers / 39,896 km/hr
t = 2.5×10^{9} hours.
T = 2.5×10^{9} hours x (1 day / 24 hours) x (1 year / 365 days) = **285,000 years**.

Problem 2 - Engineers have speculated that it might be possible to build a 'Ram Augmented Interstellar Rocket' that can reach a speed of 1,000 km/sec or greater. Using this technology, what is the maximum spacecraft travel time to each of the stars in the exoplanet list?

Answer: At a speed of 1,000 km/sec the spacecraft would travel 1 light year in 9.3 x 10^{12} km / 1,000 k/s = 300 years. Note: This speed is about 90 times faster than the Apollo-10 spacecraft for manned flight. The estimated arrival times are shown in the table below.

Name	Constellation	Distance (light years)	Travel Time (years)
Epsilon Eridani b	Eridanus	10.5	3,150
Gleise 581g	Libra	20.3	6,090
Gleise 674	Ara	14.8	4,440
Gleise 876 d	Aquarius	15	4,500
Gleise 832 b	Grus	16.1	4,830
Gleise 176	Taurus	31	9,300
Fomalhaut b	Pices Austr.	25	7,500
61 Virginis b	Virgo	28	8,400


Pleiades Cluster (Courtesy Digital Sky Survey/NASA)

During the last 40 years, a few astronomers have surveyed thousands of stars using radio telescopes in an effort to detect any signs of artificial radio signals for extraterrestrial civilizations.

The success of this listening depends on how many civilizations are *Out There* right now, beaming radio energy into space for us to over-hear, and for how long they have been doing so.

The search for an ET that is currently transmitting radio signals into space for us to over-hear can be thought of as a game of pulling a single colored ball out of a bag containing many white balls. The Milky Way galaxy contains perhaps 600 billion stars (white balls plus colored balls) to search. In this game, we will succeed if, after a limited number of draws, we hit upon just one single colored ball, representing a civilization that is currently broadcasting radio signals. Although it is impossible to appreciate the scope of such a game when 600 billion balls are involved, let's imagine a smaller game, instead.

Problem 1 - Suppose we had a bag with N=5 balls. Suppose that m= 1, 2, 3, 4 or 5 of these balls was colored (we have 5 stars in a cluster and <u>up to</u> 5 stars with intelligent civilizations):

A) Show all the possible draw sequences for N=5 objects for each selection of m.

B) Fill-in the following blanks with the corresponding probabilities for picking our first colored ball at the end of each draw:

M=1: draw 1 = 1/5; draw 2 = ----- draw 3= ---- draw 4 =---- draw 5 = ----

M=2: draw 1 = ---- ; draw 2 = 3/10; draw 3 = ---- draw 4 = ----

M=3: draw 1 = ----; draw 2 = ----; draw 3 = ----;

M=4: draw 1 = ----; draw 2 = 1/5

M=5: draw 1 = ----

C) For each M, what is the cumulative probability that you will find your first colored ball after each draw?

D) If you didn't know how many civilizations (colored balls) existed, how many stars would you have to search (out of the 5 balls in the bag) to have a at least a 50:50 chance of finding one by the second draw (star surveyed)?

Problem 1 - Answer: The possibilities are as follows, where c = star with a civilization, s = star with no civilization, and the order of the five items in each list is the order in which the stars are searched (balls are drawn):

A)

m=1	c,s,s,s,s	s,c,s,s,s	s,s,c,s,s	s,s,s,c,s	s,s,s,s,c		
m=2	c,c,s,s,s	c,s,c,s,s	c,s,s,c,s	c,s,s,s,c	s,c,c,s,s	s,c,s,c,s	s,c,s,s,c
	s,s,c,c,s	s,s,c,s,c	s,s,s,c,c				
m=3	c,c,c,s,s	c,c,s,c,s	c,c,s,s,c	c,s,c,c,s	c,s,c,s,c	c,s,s,c,c	s,c,s,c,c
	s,s,c,c,c	s,c,c,c,s	s,c,c,s,c				
m=4	c,c,c,c,s	c,c,c,s,c	c,c,s,c,c	c,s,c,c,c	s,c,c,c,c		
m=5	c,c,c,c,c						

B) Now, our success depends on detecting exactly one of these civilizations with the current draw. So, the following are the probabilities for each m:

M=1: draw 1 = 1/5; draw 2 = 1/5; draw 3=1/5; draw 4 =1/5; draw 5 = 1/5
M=2: draw 1 = 4/10; draw 2 = 3/10; draw 3 = 2/10; draw 4 = 1/10
M=3: draw 1 = 6/10; draw 2 = 3/10; draw 3 = 1/10;
M=4: draw 1 = 4/5; draw 2 = 1/5
M=5: draw 1 = 1

C) Suppose we had exactly one star with a civilization (m=1), then we need to form a probability that increases to 1.0 by the end of our sampling all 5 stars. To do this, we calculate the <u>cumulative</u> probability of success after each draw by summing the probabilities from each previous draw:

M=1: draw 1 = 1/5; draw 2 = 2/5; draw 3 = 3/5; draw 4 = 4/5; draw 5=1.0
M=2: draw 1 = 4/10; draw 2 = 7/10; draw 3 = 9/10; draw 4 = 1.0
M=3: draw 1 = 6/10; draw 2 = 9/10; draw 3 = 1.0;
M=4: draw 1 = 4/5; draw 2 = 1.0
M=5: draw 1 = 1.0

D) M=1 we need at least 3 draws; m=2 we need at least 2 draws; for m=3, 4 and 5 we only need 1 draw. Because the m=1 case has the lowest cumulative probabilities per draw, to have a 50:50 chance we would need to sample at least 3 stars just in case there is only 1 civilization (m=1).

How far away is ET from Earth?

The Milky Way has an estimated 700 billion stars scattered throughout a disk-like volume of space that is about 100,000 light years in diameter and 1,000 light years thick. Intuitively we can understand that if only a few intelligent civilizations exist today, they will on average be located at a very large distance from each other. If there are a lot of these civilizations, the average distance will be smaller.

The following problems show how to estimate this distance.

Problem 1 - what is the volume of the Milky Way in cubic light years if the formula for the volume of a circular disk is $V = \pi R^2 h$?

Problem 2 - What is the average density of stars within this volume of space in stars per cubic light year?

Problem 3 - Explain what L represents in the formula $L = \frac{1}{Density^{1/3}}$

Problem 4 - Suppose that today we lived in a very crowded Milky Way with one million intelligent civilizations able to communicate using radio technology. What would be the average distance between such civilizations if they were scattered uniformly within the volume of the Milky Way?

Problem 5 - Suppose that, pessimistically, we lived in a galaxy where there was only one other technological civilization at the present time. To two significant figures, about how far away would it be on average?

Problem 1 - what is the volume of the Milky Way in cubic light years if the formula for the volume of a circular disk is $V = \pi R^2 h$?

Answer: $V = 3.141 (50,000)^2 (1000) =$ **7.9×10^{12} cubic light years**.

Problem 2 - What is the average density of stars within this volume of space in stars per cubic light year?

Answer; Density = Number/Volume so
Density = 7×10^{11} stars / 7.9×10^{12} ly^3 = **0.089 stars/ly^3**

Problem 3 - Explain what L represents in the formula $L = \frac{1}{Density^{1/3}}$

Answer: The cube-root of density is equal to 1/length, so the reciprioCal of this quantity implies that L equals a length. This length is the average distance between the particles or objects counted in the density, so if Density = stars/Ly^3, then L is the average distance between the stars.

Problem 4 - Suppose that today we lived in a very crowded Milky Way with one million intelligent civilizations able to communicate using radio technology. What would be the average distance, to one significant figure, between such civilizations if they were scattered uniformly within the volume of the Milky Way?

Answer: Density = 1×10^6 aliens / 7.9×10^{12} ly^3 = 1.3×10^{-7} aliens/ly^3

Then, $L = 1 / (1.3 \times 10^{-7})^{1/3}$ so Distance = **200 light years**

Problem 5 - Suppose that, pessimistically, we lived in a galaxy where there was only one other technological civilization at the present time. To two significant figures, about how far away would it be from the sun on average?

Answer: Density = 2 aliens / 7.9×10^{12} ly^3 = 2.5×10^{-13} aliens/ly^3

Then, $L = 1 / (2.5 \times 10^{-13})^{1/3}$ so Distance = **16,000 light years**

Interstellar communication, the easiest way to make contact with ET, requires the evolution of radio technology. Although biological evolution is capable of innovating changes in the physical form and capabilities of an organism, there is no 'gene' for creating radio technology.

To explore the emergence of this ability, let's have a look at our own history.

Progress of technology and estimated population growth

Event	Date	Estimated Human Population	Estimated number of Elite Scientists
Speech	50,000 BC	200,000	1
Symbolism	30,000 BC	1 million	5
Wheel	10,000 BC	5 million	10
Writing	3,000 BC	15 million	50
Algebra	1,000 BC	50 million	100
Logical thinking	500 BC	100 million	200
Renaissance	1300 AD	350 million	500
Printing Press	1440 AD	400 million	600
Telescope	1609 AD	550 million	1,000
Physics	1700 AD	610 million	2,000
Electricity	1820 AD	1.1 billion	3,000
Radio Waves	1895 AD	1.6 billion	10,000
Radio Transmitter	1920 AD	2.0 billion	50,000
Radio Telescope	1937 AD	2.4 billion	100,000

Problem 1 - According to the above table, about how long did it take for modern humans (*Homo sapiens intelligens*) to develop interstellar communication technology? Answer; about 52,000 years.

Problem 2 - What was the single most important factor in enabling this technological advance and accounting for the pace of development? Answer: The growing population of 'scientifically trained' individuals.

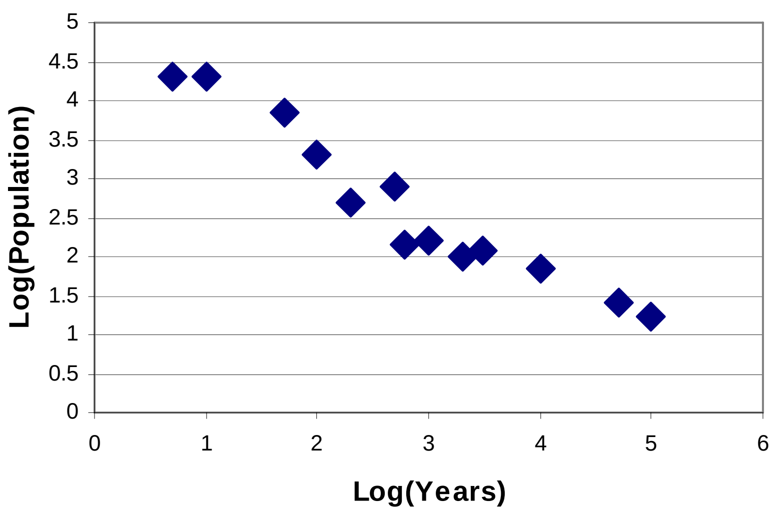
Problem 3 - Graph the interval of time between the identified events, and the approximate population size of the scientific community. Because the values span many factors of 10, plot the Log10(Years) on the horizontal axis and Log10(Population) on the vertical axis. What do you notice?

The answers to the indicated problems, and the data in the table as well, are all subject to student discussion. Students are encouraged to discuss the history of how technology emerged, the various ways in which it evolved through out human history, and how it was that new ideas emerged. Also, how the development of sophisticated communication technology (speech, writing, printing) made technological advancement possible through the recording and sharing of complex thoughts and reasoning by our ancestors so that the current generation could build on older discoveries.

Problem 1 - According to the above table, about how long did it take for modern humans (*Homo sapiens intelligens*) to develop interstellar communication technology?
Answer; **About 52,000 years**.

Problem 2 - What was the single most important factor in enabling this technological advance and accounting for the pace of development?
Answer: The growing population of 'scientifically trained' individuals.

Problem 3 - Graph the interval of time between the identified events, and the approximate population size of the scientific community. Because the values span many factors of 10, plot the Log10(Years) on the horizontal axis and Log10(Population) on the vertical axis. What do you notice?



Answer: Students may notice a number of different things, but the main thing is that, **as the size of the population increases, the time interval between discoveries decreases**. Students who are familiar with Algebra 2 concepts may realize that, because the slope is nearly constant in this graph, that a simple function that fits this behavior is the 'power-function'. For this specific case, the slope in the above graph is about m = -3/4, so the linear function is about Log(P) = -3/4 Log(T) + 5.0. We can solve this equation for P(T) to get $P(T) = 100000\, T^{-\frac{3}{4}}$

Either from natural catastrophies, or by their own hands, many civilizations have come and gone in the short history of or our own human race. It is not unreasonable to speculate that alien civilizations also suffer these same calamities. In order for two civilizations in space to be able to communicate, the timing has to be just right. It does no good at all if, after having created interstellar communication technology, two civilizations only a few light years apart, miss each other by a million years in time!

The Milky Way galaxy is about 12 billion years old. Suppose that it contained 3 advanced civilizations. Suppose, also, that each civilization survived for a whopping 4 billion years with interstellar communication technology before going extinct. What is the probability that one of these civilizations will be able to communicate with at least one other civilization, if the galaxy has produced 2 or 3 intelligent civilizations during the 12 billion years?

Let 'N' represent no civilization existing during a 4 billion year period, and 'Y' represent a civilization present during a 4 billion year period. If only one civilization exists (m=1), the possible ways in which the 12 billion year history of the galaxy can indicate the existence of the civilization are as follows: Y, N, N N, Y, N and N, N, Y. The first history string means that the civilization existed during the first 4 billion years of the galaxy's history, but not during the next 8 billion years.

Problem 1 - What are all of the possible configurations for A) m = two civilizations? B) m = three civilizations?

Problem 2 - For each value of m, what is the probability that A) exactly two civilizations will be present at the same time and able to communicate? B) exactly three civilizations will be able to communicate at the same time?

Problem 3 - Suppose you were a member of one of these three advanced civilizations. You did not know how many others there might be in the galaxy today. What would be the probability that you would have at least one other civilization to communicate with?

Problem 1 - What are all of the possible configurations for A) two civilizations? B) three civilizations?

A)

Civ 1:	Y N N	Y N N	Y N N	N Y N	N Y N	N Y N	N N Y	N N Y	N N Y
Civ 2:	Y N N	N Y N	N N Y	Y N N	N Y N	N N Y	Y N N	N Y N	N N Y

B)

Civ 1:	Y N N	Y N N	Y N N	;	Y N N	Y N N	Y N N	;	Y N N	Y N N	Y N N
Civ 2:	Y N N	Y N N	Y N N	;	N Y N	N Y N	N Y N	;	N N Y	N N Y	N N Y
Civ 3:	Y N N	N Y N	N N Y	;	Y N N	N Y N	N N Y	;	Y N N	N Y N	N N Y

Civ 1:	N Y N	N Y N	N Y N	;	N Y N	N Y N	N Y N	;	N Y N	N Y N	N Y N
Civ 2:	Y N N	Y N N	Y N N	;	N Y N	N Y N	N Y N	;	N N Y	N N Y	N N Y
Civ 3:	Y N N	N Y N	N N Y	;	Y N N	N Y N	N N Y	;	Y N N	N Y N	N N Y

Civ 1:	N N Y	N N Y	N N Y	;	N N Y	N N Y	N N Y	;	N N Y	N N Y	N N Y
Civ 2:	Y N N	Y N N	Y N N	;	N Y N	N Y N	N Y N	;	N N Y	N N Y	N N Y
Civ 3:	Y N N	N Y N	N N Y	;	Y N N	N Y N	N N Y	;	Y N N	N Y N	N N Y

Problem 2 - For each value of m, what is the probability that A) exactly two civilizations will be present at the same time and able to communicate? B) exactly three civilizations will be able to communicate at the same time?

Answer: A) For m=2 Problem 1 A) shows that there are 9 possibilities, but only 3 for which exactly two 'Ys' line up, so the probability is **P(m=2) = 1/3**.

For m=3 Problem 1 B) shows that there are 27 possibilities, but only 18 for which exactly 2 'Ys' line up, so the probability is P(m=3) = 18/27 or **P(m=3)= 2/3**.

B) Only the results for Problem 1B are relevant for which there are three cases where all three civilizations line up in time, so P(m=3) = 3/27 or **P(m=3)=1/9**.

Problem 3 - Suppose you were a member of one of these three advanced civilizations. You did not know how many others there might be in the galaxy today. What would be the probability that you would have at least one other civilization to communicate with?

Answer: The total number of possibilities for 2 and 3 civilizations is the sum of Problem 1 A and 1B or 9 + 27 = 36. The total number of cases where there are two or three civilizations able to communicate at the same time is 3 + 18 + 3 = 24, so the probability that you might have at least one other civilization to communicate with is just P = 24/36 or **P= 2/3**, not knowing how many other civilizations might be around at this time.

Interstellar Travel and Colonization

Imagine that you lived in a galaxy where the stars were separated by 1 light year, and your civilization wanted to colonize all of the stars in your galaxy. How long would this process take?

A simple mathematical model can help you arrive at a surprising answer!

Imagine that the civilization had the technology to build a colony ship, carrying 100 colonists, that could travel 1 light year in a thousand years. Suppose that their strategy was to land on a planet orbiting the star, mine for resources, and over the course of 100 years build an additional colony ship. They now have two ships capable of interstellar travel, and so they send these two ships, with 100 colonists each, to two new stars and each new colony repeats this process.

Problem 1 - For a galaxy containing 8 stars, what is the smallest number of generations of colonists needed to reach every star?

Problem 2 - How much time will elapse from start to finish in colonizing this galaxy?

Problem 3 - The Milky Way contains about 680 billion stars. The average distance between stars is about 3 light years. Suppose that the new colony requires 200 years to establish itself and manufacture a new starship, and 5000 years to travel between stars. How long, to one significant figure, will it take this civilization to colonize every star in the Milky Way?

Problem 4 - Instead of the expense of building colony ships, suppose that the civilization sent out two self-replicating, low mass space probes. When the probe reached a nearby new star, it would unfurl a large antenna and report back to Home World that it had arrived, plus return data about what it found as it encountered each planet. After reporting back, it would land on an asteroid and replicate two new probes, sending them out to two new nearby stars. Suppose that the reporting and replication process lasted 10 years and that the trip to the next star at 1% the speed of light lasted about 100 years. How long would it take for the expanding network of space probes to visit every star in the Milky Way galaxy?

Problem 1 - For a galaxy containing 8 stars, what is the smallest number of generations of colonists needed to reach every star?

Answer: There are 8 stars, and if one generation doubles the number of colonies, then three generations are needed, N = 2 x 2 x 2 = 8.

Problem 2 - How much time will elapse from start to finish in colonizing this galaxy?

Answer: Each generation lasts 100 years to establish, and 1000 years to travel, so the total time for 3 generations is 3 x (1100 years) = 3,300 years.

Problem 3 - The Milky Way contains about 680 billion stars. The average distance between stars is about 3 light years. Suppose that the new colony requires 200 years to establish itself and manufacture a new starship, and 5000 years to travel between stars. How long, to one significant figure, will it take this civilization to colonize every star in the Milky Way?

Answer: First, 680 billion = 2^N
If N = 39, we have 549 billion, if N = 40 we have 1099 billion, so there would need to be N = 39 generations of colonists. Since each colony will take 5200 to build the next generation of colony ship and make the journey to the next star, the total time needed to colonize the milky way galaxy in this way is about T = 5200 x 39 = **200,000 years!**

Problem 4 - Instead of the expense of building colony ships, suppose that the civilization sent out two self-replicating, low mass space probes. When the probe reached a nearby new star, it would unfurl a large antenna and report back to Home World that it had arrived, plus return data about what it found as it encountered each planet. After reporting back, it would land on an asteroid and replicate two new probes, sending them out to two new nearby stars. Suppose that the reporting and replication process lasted 10 years and that the trip to the next star at 1% the speed of light lasted about 100 years. How long would it take for the expanding network of space probes to visit every star in the Milky Way galaxy?

Answer: It would still take 39 generations of space probes since 680 billion stars = 2^N, and since each generation and travel leg takes 110 years, the total time would be about T = 39 x 110 years = 4,290 years!

Note: Because the diameter of the Milky Way is over 100,000 light years, as the probes venture further from Home World, the time taken for the signals to get back from the most distant probes will increase to 100,000 years!

So where are they?

Interstellar colonization of the entire galaxy, or sending self-replicating space probes to every star in the Milky Way can reach every star in only a few thousand years.

So why is it that in our solar system we seem to have no evidence that other civilizations in the Milky Way have not done this once, or even many times, in the history of Earth?

In an informal discussion in 1950, the physicist Enrico Fermi offered the following question: If a multitude of advanced extraterrestrial civilizations exists in the Milky Way, why is there no evidence for them such as radio signals, spacecraft, colonies or probes? In 1975, Michael Hart wrote an article in which he detailed the many factors that could lead to such a cosmic silence, and this is sometimes called the Fermi-Hart Paradox.

Rare Earth Hypothesis - The 'silence' we are experiencing means that the required intelligent life and technology is exceedingly rare, and that only one civilization ever exists in the lifetime of each galaxy.

We are First - We are the first such civilization, among a multitude to come over the course of the next 50 billion years.

Civilization does not last long - There could be many civilizations existing, but their lifespans are short, or their interstellar technology does not last long enough to make the long journeys.

Civilizations tend to destroy competitors - This greatly reduces the number of space-faring civilizations, because the first ones tend to destroy newcomers.

Interstellar travel and communication is harder than we think - Although it all looks 'simple' theoretically, there are unknown factors that make travel and communication much harder, and perhaps even impossible on galactic scales.

Problem 1 - How would each of these arguments affect the Drake Equation, and what probabilities might you assign to account for them?

Problem 1 - How would each of these arguments affect the Drake Equation, and what probabilities might you assign to account for them?

$$N = N^{*} f_p N_e f_l f_i f_c L$$

Astronomical Factors:
N* = the number of stars born each year in the Milky Way,
f_p is the fraction of stars with planets;
n_e is the average number of habitable planets per star;

Biological and technological Factors:
f_l is the fraction of habitable planets that have life;
f_i is the fraction that develop intelligent life;
f_c is the fraction of civilizations that develop interstellar communication, and
L is the lifetime of detectable intelligent emission.

Rare Earth Hypothesis - The 'silence' we are experiencing means that the required intelligent life and technology is exceedingly rare, and that only one civilization ever exists in the lifetime of each galaxy. **Answer: This affects f_i f_c and L**

We are First - We are the first such civilization, among a multitude to come over the course of the next 50 billion years. **Answer: This affects f_i and f_c**

Civilization does not last long - There could be many civilizations existing, but their lifespans are short, or their interstellar technology does not last long enough to make the long journeys. **Answer: This affects L**

Civilizations tend to destroy competitors - This greatly reduces the number of space-faring civilizations, because the first ones tend to destroy newcomers.
Answer: This affects f_c and L and perhaps even f_i

Interstellar travel and communication is harder than we think - Although it all looks 'simple' theoretically, there are unknown factors that make travel and communication much harder, and perhaps even impossible on galactic scales.

Answer: This affects L and f_c

Students may assign probabilities that make sense to them and discuss their various reasons.

Note: *Perhaps the Fermi paradox itself is the ultimate reason for any civilization to avoid contact with other civilizations, even if no other obstacles existed. If all civilizations regard this problem as one of allocating scarce resources for a low probability of success over hundreds of years, perhaps very few decide to 'go for broke'. We may live in a very crowded universe, but everyone is 'listening' for others to make the first move, and no one is transmitting or sending out probes, which is much more expensive.*

e•merge (ĭ-mûrj′) *v.* **e•merged, e•merg•ing. 1.**To rise up or come forth into view; appear. **2.** To come into existence. **3.** To become known or evident. [Lat. *emergere.*] **—e•mer′gence** *n.* **—e•mer′gent** *adj.*
e•mer•gen•cy (ĭ-mûr′jən-sē) *n., pl.* **-ies.** An unexpected situation or occurrence that demands immediate attention.
e•mer•i•tus (ĭ-mĕr′ĭ-təs) *adj.* Retired but retaining an honorary title: *a professor emeritus.* [Lat., p.p. of *emereri*, to earn by service.]
em•er•y (ĕm′ə-rē, ĕm′rē) *n.* A fine-grained impure corundum used for grinding and polishing. [< Gk *smuris.*]
e•met•ic (ĭ-mĕt′ĭk) *adj.* Causing vomiting. [< Gk. *emein*, to vomit.] **—e•met′ic,** *n.*
-emia *suff.* Blood: *leukemia.* [< Gk. *haima*, blood.]
em•i•grate (ĕm′ĭ-grăt′) *v.* **-grat•ed,-grat•ing.** To leave one country or region to settle in another. [Lat. *emigrare.*] **—em′i-grant** *n.* **—em′i-gra′tion** *n.*
é•mi-gré (ĕm′ĭ-grā′) *n.* An emigrant, esp. a refugee from a revolution. [Fr.]
em•i•nence (ĕm′ə-nəns) *n.* **1.** a position of great distinction or superiority. **2.** A rise or elevation of ground; hill.
em•i•nent (ĕm′ə-nənt) *adj.* **1.** Outstanding, as in reputation; distinguished. **2.** Towering above others; projecting. [< Lat. *eminēre*, to stand out.] **—em′i•nent•ly** *adv.*
em•phat•ic (ĕm-făt′ĭk) *adj.* Expressed or performed with emphasis. [< Gk. *emphatikos.*]**—em•phat′i•cal•ly** *adv.*
em•phy•se•ma (ĕm′fi-sē′mə) *n.* A disease in which the air sacs of the lungs lose their elasticity, resulting in an often severe loss of breathing ability. [< Gk. *emphusēma.*]
em•pire (ĕm′pīr′) *n.* **1.** A political unit, usu. larger than a kingdom and often comprising a number of territories or nations, ruled by a single central authority. **2.** Imperial dominion, power, or authority. [<Lat. *imperium.*]
em•pir•i•cal (ĕm-pîr′i-kəl) *adj.* Also **em•pir•ic** (-pir′ik). **1.** Based on observation or experiment. **2.** Relying on practical experience rather than theory. [<Gk. *empeirikos*, experienced.] **—em•pir′i•cal•ly** *adv.*
em•pir•i•cism (ĕm-pîr′ĭ-sĭz′əm) *n.* **1.** The view that experience, esp. of the senses, is the only source of knowledge. **2.** The employment of empirical methods, as in science.**—em•pir′i•cist** *n.*
em•place•ment (ĕm-plās′mənt) *n.* **1.** A prepared position for guns within a fortification. **2.** Placement. [Fr.]
em•ploy (ĕm-ploi′) *v.* **1.** To engage or use the services of. **2.** To put to service; use. **3.** To devote or apply (one's time or energies) to an activity. *—n.* Employment. [< Lat. *implicare*, to involve.] **—em•ploy′a•ble** *adj.*
em•ploy•ee (ĕm-ploi′ē, ĕm′ploi-ē′) *n.* Also **em•ploy•e.** One who works for another.

ă pat ā pay â care ä father ĕ pet ē be ĭ pit ī tie î pier ŏ pot ō toe ô paw, for oi noise o͝o took o͞o boot ou out th thin *th* this ŭ cut û urge yoo abuse zh vision ə about, item, edible, gallop, circus

Some of the basic principles that astronomers use to search for life in the universe can be demonstrated using a dictionary!

Imagine that each of the more than 4,000 languages spoken by humans had their own dictionary. Let's let each of these dictionaries represent a galaxy of stars, and each word represents an individual star. For this activity, we are going to search for 'life' in the English galaxy!

Step 1 - How many words are in the English Galaxy?

Step 2 - Linguists have classified each word according to its first letter from A to Z. Likewise, astronomers did the same thing, and have decided that stars similar to our sun (Class G) with classes F, G and K are likely places to search for life like our own. In the English Galaxy, how many words start with F, G or K?

Step 3 - A careful study of a small sample of F, G and K type stars finds that 10% have planets. In the English Galaxy, eliminate 9 out of every 10 pages in the F, G and K word listings and save the rest. How many words survived this culling in the English galaxy?

Step 4 - The Kepler Survey found that out of 1200 planets, about 60 are located in the Habitable Zone where temperatures are just right for liquid water. We call these Goldilocks Planets. From Step 3, discard enough words in the English Galaxy so that the words that remain match in proportion the 60 our of 1200 'Goldilocks Planets'. How many Goldilocks Words remain in the English Dictionary?

Step 5 - Look through your list of Goldilocks Words and construct a Venn Diagram that allows you to sort through the words to find those that have something to do with life or living systems. What fraction of your Goldilocks Words are 'living'?

Step 6 - In order for humans to communicate with life on other worlds, that life has to be intelligent enough to build technologies to either visit us, or communicate via radio. Are there any of your 'life' words in the English Galaxy that suggest intelligence able to communicate?

Step 1 - How many words are in the English Galaxy? Answer: A typical 'collegiate' dictionary (e.g. *Macmillian Dictionary for Students*) has about 1100 pages with 100 words per page or **110,000 words**. Other dictionaries selected by students will vary in size.

Step 2 - Linguists have classified each word according to its first letter from A to Z. Likewise, astronomers did the same thing, and have decided that stars similar to our sun (Class G) with classes F, G and K are likely places to search for life like our own. In the English Galaxy, how many words start with F, G or K? Answer: The example had 106 pages x 50 wds/page = **5,300 words**.

Step 3 - A careful study of a small sample of F, G and K type stars finds that 10% have planets. In the English Galaxy, eliminate 9 out of every 10 pages in the F, G and K word listings and save the rest. How many words survived this culling in the English galaxy? Answer: If only 10% of the pages remain, then in our example, (10%/100%) x 5300 = **530 words** remain.

Step 4 - The Kepler Survey found that out of 1200 planets, about 60 are located in the Habitable Zone where temperatures are just right for liquid water. We call these Goldilocks Planets. From Step 3, discard enough words in the English Galaxy so that the words that remain match in proportion the 60 our of 1200 'Goldilocks Planets'. How many Goldilocks Words remain in the English Dictionary? Answer: The fraction of detected planets is 60/1200 = 0.05, so since we found only 530 words with 'planets', the number of Goldilocks Worlds is just 530 x 0.05 = **27 words** out of the 110,000 we started with in our search.

Step 5 - Look through your list of Goldilocks Words and construct a Venn Diagram that allows you to sort through the words to find those that have something to do with life or living systems. What fraction of your Goldilocks Words are 'living'? Answer: With only 27 words drawn randomly from the listing for F, G and K, we might get the following list: Fair, fashion, fish, flat, flute, foe, fossil, forbid, frigate, fuel, gabby, galleon, gantry, garland, gendarme, glider, giraffe, go, gopher, grade, green, gull, guitar, karma, keno, kitty, krypton. The ones in this list that have something obvious to do with living things are ' **fashion, fish, flute, foe, fossil, frigate, gabby, galleon, gantry, gendarme, glider, giraffe, gopher, gull, guitar, keno, kitty**.' Note some of these imply a living thing (glider = a thing created by humans). So there are **17 of the 27** words that are living.

Step 6 - In order for humans to communicate with life on other worlds, that life has to be intelligent enough to build technologies to either visit us, or communicate via radio. Are there any of your 'life' words in the English Galaxy that suggest intelligence able to communicate? Answer: The words 'frigate, galleon, gantry, glider' imply a sophisticated intelligence to create them. **So only 4 'planets' out of 110,000 in the English Galaxy had 'intelligence'.**

Other similar searches could be attempted and compared to astronomers. Discussion question: What might nouns, verbs, adjectives represent in the astronomical universe?

Appendix A - Astrobiology Resources

NASA Astrobiology Institute

http://astrobiology.nasa.gov/nai/

SETI@Home

http://setiathome.berkeley.edu/

Kepler Website

http://kepler.nasa.gov/

Exoplanet Data Explorer

http://exoplanets.org/

NASA Planet Quest

http://planetquest.jpl.nasa.gov/

The Planetary Society

http://www.planetary.org/home/

The Astrobiology Web

http://www.astrobiology.com/

The Virtual Museum of Bacteria (Extremophiles)

http://bacteriamuseum.org/cms/Evolution/extremophiles.html

Bibliography: (http://www.hq.nasa.gov/office/hqlibrary/pathfinders/astro.htm)

The Astrobiology Primer: An Outline of General Knowledge – Version 1, 2006, Bennett, Jeffrey, Shostak, Seth, and Jakosky, Bruce. *Life in the Universe*. San Francisco: Addison Wesley, 2003.

First Contact: The Search for Extraterrestrial Intelligence, Bova, Ben and Preiss, Byron (eds.). New York: NAL Books, 1990.

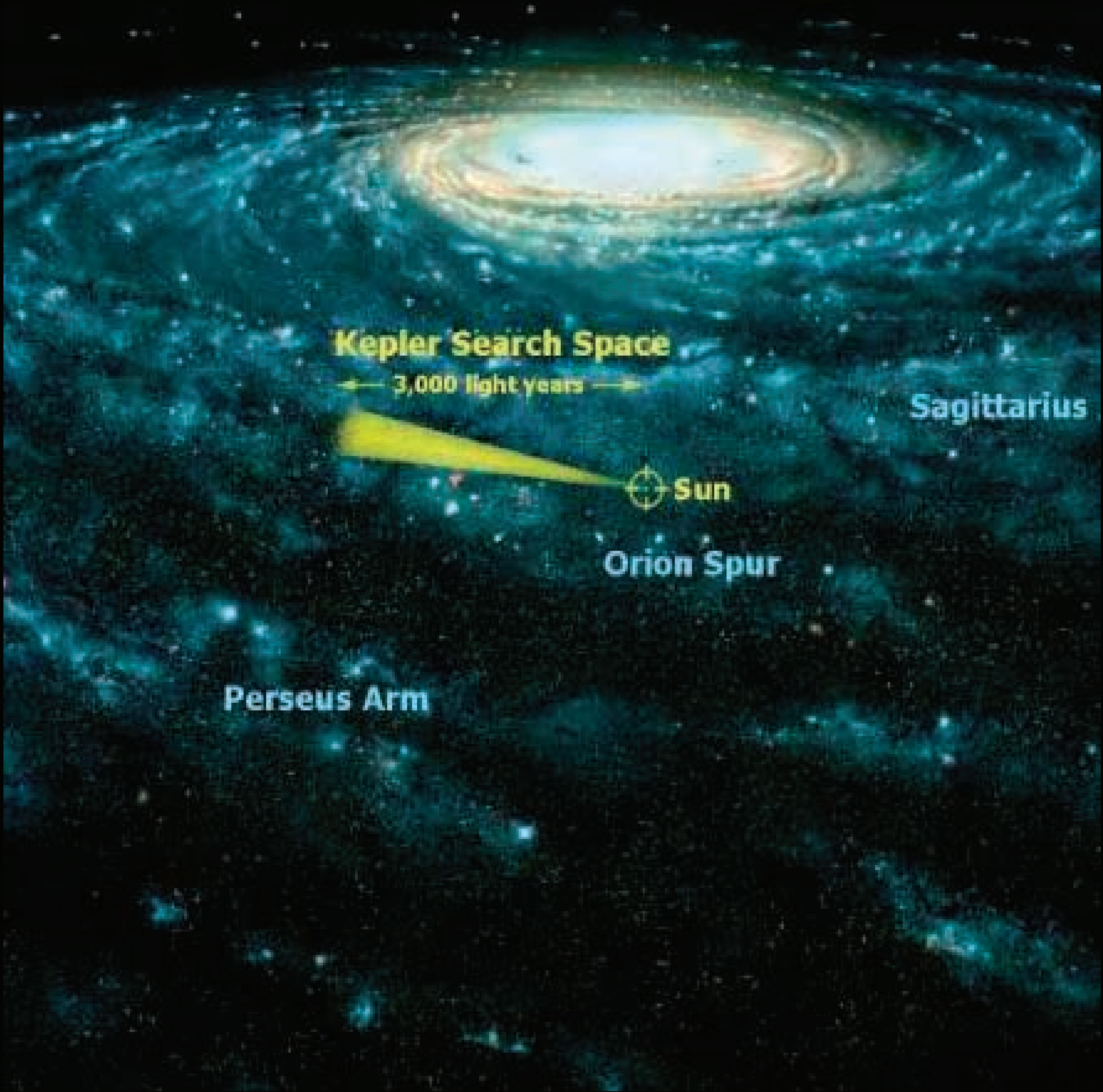
Life Everywhere: The Maverick Science of Astrobiology, Darling, D. , Cambridge, UK; New York: Basic Books, 2001.

Genesis: The Scientific Quest for Life's Origin, Hazen, Robert M. Washington, DC: Joseph Henry Press, 2005.

Science, Society, and the Search for Life in the Universe, Jakosky, Bruce M. Tucson: University of Arizona Press, 2006.

Lemonick, Michael D. *Other Worlds: The Search for Life in the Universe*. New York: Simon & Schuster, 1998.

Sullivan, Woodruff T. and Baross, John S. (Eds.). *Planets and Life: The Emerging Science of Astrobiology*. Cambridge, UK: Cambridge University Press, 2007.



National Aeronautics and Space Administration

Space Math @ NASA
Goddard Spaceflight Center
Greenbelt, Maryland 20771
spacemath.gsfc.nasa.gov

www.nasa.gov